MA262 — EXAM II — FALL 2019 — NOVEMBER 19, 2019
TEST NUMBER 11 — GREEN

INSTRUCTIONS:

1. Do not open the exam booklet until you are instructed to do so.
2. Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.
3. *Mark your test number on your scantron*
4. Once you are allowed to open the exam, make sure you have a complete test. There are 7 different test pages including this cover page.
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers on this test booklet.
6. The exam has 11 problems and each one is worth 9 points and everyone gets one point. The maximum possible score is 100 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 40 minutes, you must wait until the end of the exam period to leave the room.
9. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

DON'T BE A CHEATER:

1. Do not give, seek or obtain any kind of help from anyone to answer questions on this exam. If you have doubts, consult only your instructor.
2. Do not look at the exam or scantron of another student.
3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, given it to your instructor and left the room.
5. Do not consult notes or books.
6. Do not handle phones or cameras, calculators or any electronic device until after you have finished your exam, given it to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ____________________

STUDENT SIGNATURE: ____________________

STUDENT ID NUMBER: ____________________

SECTION NUMBER AND RECITATION INSTRUCTOR: ____________________

1. Let $\mathbf{A}$ be an $n \times n$ nonsingular matrix. Which of the following statements must be true?

(i) $\det \mathbf{A} = 0$.
(ii) $\text{rank}(\mathbf{A}) = n$.
(iii) $\mathbf{Ax} = \mathbf{0}$ has infinitely many solutions.
(iv) $\mathbf{Ax} = \mathbf{b}$ has a unique solution for every vector $\mathbf{b} \in \mathbb{R}^n$.
(v) $\mathbf{A}$ must be row equivalent to the $n \times n$ identity matrix $\mathbf{I}_n$.

A. (i) and (iii) only
B. (ii) and (iv) only
C. (i), (iv) and (v)
D. (i), (iii) and (v)
E. (ii), (iv) and (v)

2. Consider the initial value problem: $t(t-10)y'' + y' - \dfrac{1}{t-3}y = \ln(t-5)$, $y(6) = 0$, $y'(6) = 1$.
Find the largest interval for which the above initial value problem has a unique solution.

A. $(0, 5)$
B. $(0, 10)$
C. $(5, +\infty)$
D. $(10, +\infty)$
E. $(5, 10)$

3. Which of the following subset S is a subspace of V?

(i) $V = \mathbb{R}^3$ and S is the set of vectors (x, y, z) satisfying $x + 2y - 3z = 0$.
(ii) $V = M_2(\mathbb{R})$ and S is the set of 2×2 matrices with determinant $\neq 0$.
(iii) $V = P_2$ and S is the set of polynomials of the form $ax^2 - bx$, where $a, b \in \mathbb{R}$.
(iv) $V = M_n(\mathbb{R})$ and S is the set of $n \times n$ nonsymmetric matrices.

A. (i) and (iii) only
B. (i) and (iv) only
C. (ii) and (iii) only
D. (i) (iii) and (iv)
E. (i) (ii) and (iii)

4. Determine the general solution to $(D + 1)(D - 1)^2(D^2 + 2D + 2)y = 0$.

A. $c_1e^{-x} + c_2e^{x} + e^{-x}(c_3 \cos x + c_4 \sin x)$
B. $c_1e^{-x} + c_2e^{x} + c_3xe^{x} + e^{-x}(c_4 \cos x + c_5 \sin x)$
C. $c_1e^{-x} + c_2e^{x} + e^{x}(c_3 \cos x + c_4 \sin x)$
D. $c_1e^{-x} + c_2xe^{-x} + c_3e^{x} + e^{-x}(c_4 \cos x + c_5 \sin x)$
E. $c_1e^{-x} + c_2e^{x} + c_3xe^{x} + e^{x}(c_4 \cos x + c_5 \sin x)$

5. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 2 & 1 \\ 2 & 4 & 0 & 2 & -2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$, which of the following set is a basis of the column space of A?

A. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \\ 1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ -2 \\ 2 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \\ 1 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ -2 \\ 2 \end{bmatrix} \right\}$

6. The general solution of $y^{(4)} - 8y'' + 16y = 0$ is

A. $c_1e^{2x} + c_2xe^{2x} + c_3e^{-2x} + c_4xe^{-2x}$
B. $c_1e^{2x} + c_2e^{-2x}$
C. $c_1e^{4x} + c_2e^{-4x}$
D. $c_1e^{2x} + c_2e^{-2x} + c_3$
E. $c_1e^{4x} + c_2xe^{4x} + c_3e^{-4x} + c_4xe^{-4x}$

7. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$, which of the following set is a basis of the null space of A?

A. $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$

8. Let $y(t)$ be the solution to the initial value problem $y'' + 3y' - 4y = 6e^{2t}$, $y(0) = 2$, $y'(0) = 3$, find $y(1)$.

A. e
B. e^2
C. $e + e^2$
D. $e - e^2$
E. $2e - e^2$

9. Given that $y_1(t) = t$ is a solution to $t^2y'' - ty' + y = 0$, $t > 0$, find a second linearly independent solution $y_2(t)$.

A. t^2
B. $t \ln t$
C. $\ln t$
D. $t^2 \ln t$
E. te^t

10. Let $\lambda = 3$ be an eigenvalue of $A = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 4 & 1 \\ 0 & 0 & 3 \end{bmatrix}$. Then the geometric multiplicity of $\lambda = 3$ is

A. 0
B. 1
C. 2
D. 3
E. 4

11. Consider a spring-mass system whose motion is governed by $y''+y = 4\sin(t)$, $y(0) = 2$, $y'(0) = 0$. Find the solution of the above initial value problem.

A. $y(t) = 2\cos(t) - \sin(t)$
B. $y(t) = \cos(t) + 2\sin(t)$
C. $y(t) = \cos(t) + \sin(t) - t\cos(t)$
D. $y(t) = 2\cos(t) + 2\sin(t) - 2t\cos(t)$
E. $y(t) = 2\cos(t) - 2\sin(t) + t\cos(t)$