1. Find a linear homogeneous constant-coefficient equation with the given general solution.

\[(A + Bx + Cx^2) e^{3x}\]

Choose the correct answer below.

- **A.** \(y^{(4)} - 12y^{(3)} + 54y'' - 108y' + 81y = 0\)
- **B.** \(y' + 3y = 0\)
- **C.** \(y^{(3)} - 9y'' + 27y' - 27y = 0\)
- **D.** \(y'' - 6y' + 9y = 0\)

2. Three vectors \(\mathbf{v}_1, \mathbf{v}_2,\) and \(\mathbf{v}_3\) are given. If they are linearly independent, show this; otherwise, find a nontrivial linear combination of them that is equal to the zero vector.

\[
\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 3 \\ -2 \\ 11 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 5 \\ -7 \\ 17 \end{bmatrix}
\]

Select the correct answer below, and fill in the answer box(es) to complete your choice.

- **A.**

  The vectors \(\mathbf{v}_1, \mathbf{v}_2,\) and \(\mathbf{v}_3\) are linearly independent. The augmented matrix \(\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & 0 \end{bmatrix}\) has an echelon for

  (Type an integer or simplified fraction for each matrix element.)

- **B.** The vectors \(\mathbf{v}_1, \mathbf{v}_2,\) and \(\mathbf{v}_3\) are linearly dependent, because \(4\mathbf{v}_1 + (\underline{\quad}) \mathbf{v}_2 + (\underline{\quad}) \mathbf{v}_3 = 0.\)

  (Type integers or fractions.)
3. Find both a basis for the row space and a basis for the column space of the given matrix \( A \).

\[
A = \begin{bmatrix}
  4 & 2 & -5 & 7 \\
 12 & 1 & 6 & 18 \\
  4 & -3 & 16 & 4 \\
  8 & -11 & 53 & 5
\end{bmatrix}
\]

A basis for the row space is \( \left\{ \begin{bmatrix} 4 & 2 & -5 & 7 \end{bmatrix}, \begin{bmatrix} 0 & -5 & 21 & -3 \end{bmatrix} \right\} \).
(Use a comma to separate matrices as needed.)

A basis for the column space is \( \left\{ \begin{bmatrix} 4 \\ 12 \\ 4 \\ 8 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \\ -11 \end{bmatrix} \right\} \).
(Use a comma to separate matrices as needed.)

4. Find a general solution to the differential equation given below. Primes denote derivatives with respect to \( t \).

\[ 56y'' - 5y' - 6y = 0 \]

A general solution is \( y(t) = c_1 e^{\frac{3}{8}t} + c_2 e^{-\frac{2}{7}t} \).

5. Let a subset \( W \) be the set of all vectors in \( \mathbb{R}^4 \) such that \( x_1 + 4x_2 + 5x_3 + 6x_4 = 0 \). Apply the theorem for conditions for a subspace to determine whether or not \( W \) is a subspace of \( \mathbb{R}^4 \).

According to the theorem of conditions for a subspace, the nonempty subset \( W \) of the vector space \( V \) is a subspace of \( V \) if and only if it satisfies the following two conditions:

(i) If \( u \) and \( v \) are vectors in \( W \), then \( u + v \) is also in \( W \).
(ii) If \( u \) is in \( W \) and \( c \) is a scalar, then the vector \( cu \) is also in \( W \).

Select the correct choice below.

- **A.** \( W \) is not a subspace of \( \mathbb{R}^4 \) because condition (ii) fails while condition (i) is satisfied.
- **B.** \( W \) is a subspace of \( \mathbb{R}^4 \) because it satisfies both of the conditions.
- **C.** \( W \) is not a subspace of \( \mathbb{R}^4 \) because both conditions (i) and (ii) fail.
- **D.** \( W \) is not a subspace of \( \mathbb{R}^4 \) because condition (i) fails while condition (ii) is satisfied.
6. Determine whether the given vectors \( \mathbf{u} \) and \( \mathbf{v} \) are linearly dependent or linearly independent.

\[
\mathbf{u} = \begin{bmatrix} -6 \\ 6 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}
\]

Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- A. The vectors are linearly dependent because \( \mathbf{v} \) is a scalar multiple of \( \mathbf{u} \), specifically \( \mathbf{v} = \underline{\text{}}\mathbf{u} \).
  (Type an integer or a fraction.)

- B. The vectors are linearly independent because \( \mathbf{v} \) is a scalar multiple of \( \mathbf{u} \), specifically \( \mathbf{v} = \underline{\text{}}\mathbf{u} \).
  (Type an integer or a fraction.)

- C. The vectors are linearly dependent because the only solution to the vector equation \( a\mathbf{u} + b\mathbf{v} = \mathbf{0} \) is \( a = \underline{\text{}} \).
  (Type integers or fractions.)

- D. The vectors are linearly independent because the only solution to the vector equation \( a\mathbf{u} + b\mathbf{v} = \mathbf{0} \) is \( a = \underline{\text{}} \).
  (Type integers or fractions.)

7. If \( A \) is a 3x5 matrix. Which of the following statements are correct?

I) There is only one 5x1 vector \( \mathbf{X} \) such that \( A\mathbf{X} = \mathbf{0} \).

II) There are infinitely many 5x1 vectors \( \mathbf{X} \) such that \( A\mathbf{X} = \mathbf{0} \).

III) If \( \mathbf{b} \) is an arbitrary 3x1 vector, one can always find a 5x1 vector \( \mathbf{X} \) which satisfies the system \( A\mathbf{X} = \mathbf{b} \).

IV) If \( \mathbf{b} \) is a 3x1 vector and \( \mathbf{X} \) is a 5x1 vector such that \( A\mathbf{X} = \mathbf{b} \), then there are infinitely many 5x1 vectors \( \mathbf{Y} \) that satisfy \( A\mathbf{Y} = \mathbf{b} \).

- A. Only I is true, II, III and IV are false
- B. I and III are true, II and IV are false
- C. II and IV are true, I and III are false
- D. I and IV are true, II and III are false
- E. Only II is true, I, III and IV are false

8. If \( A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 4 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} \) and if \( C = \begin{bmatrix} c_{ij} \end{bmatrix} \) is the matrix such that \( A\mathbf{B} \), then which of the following statements is correct about the matrix \( C \) and its element \( c_{ij} \) which is on the \( i \)-th row and \( j \)-th column?

- A. \( C \) is a 2x3 matrix and \( c_{11} = 2 \)
- B. \( C \) is a 2x3 matrix and \( c_{22} = -19 \)
- C. \( C \) is a 3x2 matrix and \( c_{11} = 0 \)
- D. \( C \) is a 2x3 matrix and \( c_{12} = -1 \)
- E. \( C \) is a 2x3 matrix and \( c_{13} = 6 \)
9. Which of the following are true statements about the solutions of the system

\[ \begin{align*}
  x_1 - x_2 + 3x_3 &= 1 \\
  x_1 + (b - 1)x_2 + x_3 &= 2 \\
  x_1 + x_2 + x_3 &= b,
\end{align*} \]

where b is a constant?

I) The system has no solutions if \( b=2 \).

II) The system has only one solution if \( b \) is not equal to 2.

III) The system has infinitely many solutions if \( b=2 \).

IV) The system has no solutions if \( b=4 \).

- A. III and IV are true, I and II are false
- B. I and II are true, III and IV are false
- C. II and III are true, I and IV are false
- D. I and IV are true, II and III are false
- E. II is true, I, III and IV are false

10. Find a basis for the indicated subspace of \( \mathbb{R}^3 \).

The line of intersection of the planes with equations \( x - 5y + 3z = 0 \) and \( y = z \).

A basis for the indicated subspace of \( \mathbb{R}^3 \) is \[ \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \].

(Use a comma to separate vectors as needed.)

11. If \( y(x) \) is the solution of the following initial value problem

\[ \begin{align*}
  y''(x) - 5y'(x) + 6y(x) &= 0 \\
  y(0) &= 3 \quad \text{and} \quad y'(0) = 7
\end{align*} \]

then \( y(\ln(2)) \) (that is, the value of \( y(x) \) when \( x = \ln(2) \)) is equal to

- A. 12
- B. 10
- C. 8
- D. 18
- E. 16