MA262 — EXAM II — FALL 2022 — NOVEMBER 10, 2022
TEST NUMBER 01

INSTRUCTIONS:

1. **DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO**
2. Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.
3. **MAKE SURE YOU WRITE YOUR 10 DIGIT ID # AND YOUR TEST NUMBER ON YOUR SCANTRON. THIS IS TEST NUMBER 01.**
4. **USE FOUR DIGITS TO ENTER YOUR RECITATION SECTION, FOR EXAMPLE 0256.**
5. Once you are allowed to open the exam, make sure you have a complete test. **There are 9 different test pages including this cover page.**
6. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. No extra paper is allowed. Circle your answers on this test booklet.
7. There are eleven problems, each problem is worth 9 points and everyone gets one point. The maximum possible score is 100 points. No partial credit.
8. After you finish the exam, hand in your scantron and your test booklet to your professor, your TA or one of the proctors.

RULES REGARDING ACADEMIC DISHONESTY:

1. Do not leave the exam room during the first 20 minutes of the exam.
2. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
3. Do not seek or obtain any kind of help from anyone to answer questions on this exam.
4. Do not look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes, books, calculators. Do not handle phones or cameras, or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
6. After time is called, the students have to put down all writing instruments and remain in their seats, while the proctors will collect the scantrons and the exams.
7. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ____________________ ID NUMBER: ____________________

SIGNATURE: ____

RECITATION SEC. NUMBER ________________ TA's NAME: ________________

1. Let $A = \begin{bmatrix} 1 & -3 & 0 & -5 \\ -1 & 4 & 1 & 7 \\ 2 & 1 & 7 & 4 \\ -2 & 2 & -4 & 2 \end{bmatrix}$, which of the following sets is a basis of the column space of A?

A. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 7 \\ -4 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \\ 2 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 7 \\ 2 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \\ 4 \\ 2 \end{bmatrix} \right\}$

2. Let $A = \begin{bmatrix} 1 & -3 & 0 & -5 \\ -1 & 4 & 1 & 7 \\ 2 & 1 & 7 & 4 \\ -2 & 2 & -4 & 2 \end{bmatrix}$, which is the same matrix of problem 1. Which of the following sets is a basis of the row space of A, if we write them as column vectors?

A. $\left\{ \begin{bmatrix} 1 \\ -3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ -3 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 3 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ -3 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ -3 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \\ 4 \\ 2 \end{bmatrix} \right\}$

3. Which of the following statements are true?

(I) If the reduced row echelon form of a $3{\times}5$ matrix A is $\operatorname{rref}(A) = \begin{bmatrix} 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$, then the dimension of the Solution Space for $A\mathbf{x} = \mathbf{0}$ must be 3.

(II) If two $m \times n$ matrices A and B have the same reduced row echelon form, then $\operatorname{rank}(A) = \operatorname{rank}(B)$.

(III) If W is the set of all vectors in $\mathbb{R}^2$ of the form $\begin{bmatrix} x \\ x^2 \end{bmatrix}$, then W is a subspace of $\mathbb{R}^2$.

(IV) The set $W = \left\{ t \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} : \ t, s \text{ are real numbers} \right\}$ is a subspace of $\mathbb{R}^3$.

A. Only **(II)** and **(IV)** are true

B. Only **(I)** and **(II)** are true

C. Only **(I)**, **(II)** and **(IV)** are true

D. Only **(III)** and **(IV)** are true

E. Only **(II)** and **(III)** are true

4. Suppose A is a 4×7 matrix whose reduced row echelon form is

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Which of the following statements are true?

(I) $\text{rank}(A) = 4.$

(II) The dimension of the *Column Space* of A must be 4.

(III) The dimension of the *Solution Space* for $A\mathbf{x} = \mathbf{0}$ must be 4.

(IV) A basis for the *Row Space* of A is

$$\Big\{ \begin{bmatrix} 1 & 0 & 1 & 0 & -1 & 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & 1 & 2 & -3 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \Big\}.$$

A. Only **(IV)** is true

B. Only **(II)** and **(III)** are true

C. Only **(II), (III)** and **(IV)** are true

D. Only **(III)** and **(IV)** are true

E. All four statements are true

5. Find all values of k such that $\begin{bmatrix} 2 \\ k \\ -2 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 3 \\ k \end{bmatrix}$.

A. $k = -1$ and $k = 4$

B. $k = 4$ and $k = 6$

C. $k = 1$ and $k = 2$

D. $k = 0$ and $k = 4$

E. $k = -1$ and $k = 0$

6. Determine the general solution of the equation

$$(D-1)(D+1)^2(D^2-2D+2)y = 0$$

(this means that its characteristic polynomial is $p(r) = (r-1)(r+1)^2(r^2-2r+2)$).

A. $C_1e^x + C_2e^{-x} + e^x(C_3\cos x + C_4\sin x)$

B. $C_1e^x + C_2e^{-x} + e^{-x}(C_3\cos x + C_4\sin x)$

C. $C_1e^x + C_2e^{-x} + C_3xe^{-x} + e^x(C_4\cos x + C_5\sin x)$

D. $C_1e^x + C_2xe^x + C_3e^x + e^x(C_4\cos x + C_5\sin x)$

E. $C_1e^x + C_2e^{-x} + C_3xe^{-x} + e^{-x}(C_4\cos x + C_5\sin x)$

7. Determine k such that $\begin{vmatrix} 2 & 1 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 2 & 1 & 1 & -1 \\ 4 & 2 & 2 & 2 \end{vmatrix} + \begin{vmatrix} k & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{vmatrix} = 0$. (Recall that if A is a square matrix $|A|$ denotes the determinant of A. So this is the sum of the determinants of the two matrices.)

A. $k = 2$

B. $k = -2$

C. $k = 8$

D. $k = 0$

E. $k = -4$

8. Find two values of m such that $y(x) = x^m$ is a solution of $2x^2y'' + xy' - 3y = 0$, $x > 0$.

A. $m = -1$ and $m = \dfrac{5}{2}$

B. $m = -1$ and $m = \dfrac{3}{2}$

C. $m = 1$ and $m = \dfrac{2}{3}$

D. $m = -2$ and $m = -\dfrac{3}{2}$

E. $m = -2$ and $m = \dfrac{2}{3}$

9. Let $x(t)$ satisfy the initial value problem

$$\begin{aligned} &x'' + 4x' + 5x = 0, \\ &x(0) = 1, \quad x'(0) = -2. \end{aligned}$$

Then $x(2\pi)$, that is $x(t)$ evaluated at $t = 2\pi$, is equal to

A. $x(2\pi) = e^{4\pi}$

B. $x(2\pi) = e^{-4\pi}$

C. $x(2\pi) = e^{-2\pi}$

D. $x(2\pi) = e^{6\pi}$

E. $x(2\pi) = e^{-6\pi}$

10. According to the method of undetermined coefficients, to find a particular solution of the differential equation

$$y'' - 2y' + y = 3xe^x + xe^x \sin x + e^{2x}$$

one must try a solution of the form

A. $y_p(x) = C_1 e^{2x} + (D_1 + D_2 x)e^x + (E_1 + E_2 x)e^x \sin x + (F_1 + F_2 x)e^x \cos x$

B. $y_p(x) = C_1 e^{2x} + x(D_1 + D_2 x)e^x + x(E_1 + E_2 x)e^x \sin x + x(F_1 + F_2 x)e^x \cos x$

C. $y_p(x) = C_1 e^{2x} + x^2(D_1 + D_2 x)e^x + x^2(E_1 + E_2 x)e^x \sin x + x^2(F_1 + F_2 x)e^x \cos x$

D. $y_p(x) = C_1 e^{2x} + x^2(D_1 + D_2 x)e^x + (E_1 + E_2 x)e^x \sin x + (F_1 + F_2 x)e^x \cos x$

E. $y_p(x) = C_1 e^{2x} + x^3(D_1 + D_2 x)e^x + (E_1 + E_2 x)e^x \sin x + (F_1 + F_2 x)e^x \cos x$

11. Let $y(x)$ satisfy the initial value problem

$$y'' - y = -x^2,$$
$$y(0) = 4 \;\; y'(0) = 0.$$

Then $y(1)$ ($y(x)$ evaluated at $x = 1$) is equal to

A. $y(1) = e + \frac{1}{e} + 3$

B. $y(1) = e + \frac{2}{e} + 1$

C. $y(1) = 2e + \frac{1}{e} - 3$

D. $y(1) = e + \frac{3}{e} + 2$

E. $y(1) = 3e + \frac{2}{e} - 1$