

**MA262 — EXAM 2 — SPRING 2016 — APRIL 04, 2016**  
**TEST NUMBER 01**

**INSTRUCTIONS:**

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
6. Each problem is worth 10 points. The maximum possible score is 100 points. No partial credit.
7. Use a # 2 pencil to fill in the answers on your scantron
8. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

**RULES REGARDING ACADEMIC DISHONESTY:**

1. Do not leave the exam room during the first 20 minutes of the exam.
2. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
3. Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
4. Do not look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes, books, calculators.
6. Do not handle phones or cameras, or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: \_\_\_\_\_

STUDENT SIGNATURE: \_\_\_\_\_

STUDENT ID NUMBER: \_\_\_\_\_

SECTION NUMBER \_\_\_\_\_

RECITATION INSTRUCTOR: \_\_\_\_\_

1. Let  $W$  be the subspace of  $\mathbb{R}^4$  which consists of the vectors  $\mathbf{v} = (x_1, x_2, x_3, x_4)$  such that  $3x_1 + 4x_2 + x_3 - x_4 = 0$ . Which of the following is a basis of  $W$ ?

A.  $(1, 0, 0, 3), (0, 1, 0, 4), (0, 0, 1, 1)$

B.  $(0, 1, 1, 1), (0, 1, 0, 4), (0, 0, 1, 1)$

C.  $(1, 0, 0, 3), (0, 1, 0, 1), (1, 1, 0, 0)$

D.  $(0, 1, 0, 4), (1, 1, 0, 0), (0, 1, 3, 1)$

E.  $(1, 2, 0, 1), (1, 1, 0, 0), (0, 1, 0, 0)$

2. Let  $V = M_3(\mathbb{R})$  be the vector space of all  $3 \times 3$  matrices with real entries. Let  $S$  be the subspace of  $V$  which consists of elements of the form  $A = [a_{ij}]$  such that  $a_{11} = a_{22} = a_{33} = 0$  and  $a_{ij} = a_{ji}$ . The dimension of  $S$  is equal to

A. 2

B. 3

C. 4

D. 5

E. 6

3. Let  $A$  be a  $7 \times 10$  matrix of rank 5. Let  $x$  be the dimension of the null space of  $A$ , and  $y$  be the dimension of the null space of  $A^T$ , the transpose of  $A$ . What are  $x$  and  $y$ ?

A.  $x = 2$  and  $y = 5$

B.  $x = 2$  and  $y = 4$

C.  $x = 4$  and  $y = 3$

D.  $x = 5$  and  $y = 4$

E.  $x = 5$  and  $y = 2$

4. Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be the linear transformation given by  $Tx = Ax$ , where  $A$  is the matrix

$$\begin{pmatrix} 1 & 1 & 4 & 0 \\ 2 & 1 & 5 & -1 \\ 1 & 0 & 1 & -1 \end{pmatrix}.$$

Which of the following is a basis of the kernel of  $T$ ?

A.  $(0, 3, 1, 0)^T, (1, 0, 0, 1)^T$

B.  $(2, 4, 1, 0)^T, (3, -1, 0, 1)^T$

C.  $(-1, -3, 1, 0)^T, (1, -1, 0, 1)^T$

D.  $(1, 2, -1, -3)^T, (-1, 2, 0, -1)^T$

E.  $(2, 1, 1, 0)^T, (-1, 1, 0, 1)^T$

5. Consider the matrix  $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 5 & -3 \end{bmatrix}$ . Determine which of the following statements are TRUE.

- (i)  $A$  has three real eigenvalues
- (ii)  $A$  has three linearly independent eigenvectors.
- (iii)  $A$  is defective

A. (i) and (ii) only

B. (i) and (iii) only

C. (ii) and (iii) only

D. (i) only

E. (ii) only

6. Find the general solution to the differential equation  $(D - 4)(D^2 + 2D + 5)y = 0$ .

A.  $y(x) = c_1e^{4x} + c_2e^x \cos(2x) + c_3e^x \sin(2x)$

B.  $y(x) = c_1e^{4x} + c_2e^x \cos(4x) + c_3e^x \sin(4x)$

C.  $y(x) = c_1e^{4x} + c_2e^{-x} \cos(4x) + c_3e^{-x} \sin(4x)$

D.  $y(x) = c_1e^{4x} + c_2e^{-x} \cos(2x) + c_3e^{-x} \sin(2x)$

E.  $y(x) = c_1e^{4x} + c_2 \cos(2x) + c_3 \sin(2x)$

7. Find the general solution to the differential equation

$$y^{(4)} - 8y'' + 16y = 0.$$

A.  $y = c_1e^{2x} + c_2e^{-2x}$

B.  $y = c_1xe^{2x} + c_2xe^{-2x}$

C.  $y = c_1e^{2x} + c_2e^{-2x} + c_3xe^{2x} + c_4xe^{-2x}$

D.  $y = c_1xe^{2x} + c_2xe^{-2x} + c_3x^2e^{2x} + c_4x^2e^{-2x}$

E.  $y = c_1 \cos 2x + c_2 \sin 2x + c_3x \cos 2x + c_4x \sin 2x$

8. Let  $y(x)$  satisfy

$$y'' + 9y' + 18 = 0, \quad y(0) = 0, \quad y'(0) = 3.$$

Then  $y\left(\frac{1}{3} \log 3\right) = ?$

A.  $y\left(\frac{1}{3} \log 3\right) = \frac{2}{9}$

B.  $y\left(\frac{1}{3} \log 3\right) = \frac{2}{3}$

C.  $y\left(\frac{1}{3} \log 3\right) = \frac{3}{4}$

D.  $y\left(\frac{1}{3} \log 3\right) = \frac{1}{3}$

E.  $y\left(\frac{1}{3} \log 3\right) = \frac{1}{4}$

9. A particular solution of  $y'' + y = \sin x$  has the form:

A.  $y = A \sin x + B \cos x$

B.  $y = Ax \sin x + Bx \cos x$

C.  $y = A \sin x$

D.  $y = B \cos x$

E. None of the above

10. An annihilator of  $e^x - x$  is:

A.  $D^2(D - 1)$

B.  $D(D - 1)$

C.  $(D + 1)^2$

D.  $D^2 - 1$

E. None of the above.