MA 262, Spring 2019, Midterm 2
GREEN 01

INSTRUCTIONS

1. **Switch off your phone upon entering the exam room.**
2. **Do not open the exam booklet until you are instructed to do so.**
3. **Before you open the booklet, fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
4. **MARK YOUR TEST NUMBER ON THE SCANTRON**
5. Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages with a total of 12 problems, plus this cover page.
6. Do any necessary work for each problem on the space provided, or on the back of the pages of this booklet. Circle your answers in the booklet.
7. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY

1. Do not leave the exam during the first 20 minutes of the exam.
2. No talking. Do not seek or obtain any kind of help from anyone to answer the problems on the exam. If you need assistance, consult an instructor.
3. Do not look at the exam of another student. You may not compare answers with other students until your exam is finished and turned in, and then only after you have left the room.
4. Your bags must be closed throughout the exam period.
5. Notes, books, calculators and phones must be in your bags and cannot be used.
6. Do not handle phones or cameras or any other electronic device until you have finished and turned in your exam, and then only if you have left the room.
7. When time is called, all students must put down their writing instruments immediately.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME ______________________________

STUDENT SIGNATURE ______________________________

STUDENT PUID ______________________________

SECTION NUMBER ______________________________

RECITATION INSTRUCTOR ______________________________

1. (8 points) Consider the following two matrices:

$$A = \begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 1 & 3 \\ -2 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

Then $\det(A^3 B^{-1}) =$

A. 6

B. $\frac{16}{5}$

C. $-\frac{8}{3}$

D. -3

E. $\frac{4}{9}$

2. (8 points) Find all values of k so that $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ k \\ 1 \end{bmatrix}$ form a basis for $\mathbb{R}^3$.

A. $k = 0$

B. $k = 1$

C. $k = 2$

D. $k \neq 1$

E. $k \neq 2$

3. (8 points) $W = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : 2a + 2b + c = d \right\}$ is a subspace of $\mathbb{R}^4$. Find $\dim W$.

A. 0

B. 1

C. 2

D. 3

E. 4

4. (8 points) Let A be a 4×5 matrix and B be its reduced row echelon form. Which of the following statement(s) must be true?

(i) The pivot columns of B form a basis for Col A.

(ii) The nonzero rows of B form a basis for Row A.

(iii) Rank A could be any integer from 0 to 5.

A. (i) only

B. (ii) only

C. (iii) only

D. (i), (ii), and (iii)

E. none of the above

5. (8 points) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 6 & 4 \end{bmatrix}$. Find the product of all the eigenvalues of A.

A. -16

B. -8

C. 0

D. 8

E. 16

6. (8 points) Let $A = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$. Then $\begin{bmatrix} -\mathbf{i} \\ 1 \end{bmatrix}$

A. is an eigenvector corresponding to eigenvalue 0.

B. is an eigenvector corresponding to eigenvalue -3.

C. is an eigenvector corresponding to eigenvalue 3.

D. is an eigenvector corresponding to eigenvalue $-3\mathbf{i}$.

E. is an eigenvector corresponding to eigenvalue $3\mathbf{i}$.

7. (8 points) A particular solution of $y''(t) - 9y(t) = 2e^{3t} + \cos 3t$ has the form

A. $Ae^{3t} + B\cos 3t$

B. $Ae^{3t} + B\cos 3t + C\sin 3t$

C. $Ate^{3t} + B\cos 3t + C\sin 3t$

D. $Ae^{3t} + Bt\cos 3t + Ct\sin 3t$

E. $Ae^{3t} + (Bt + C)\cos 3t + (Dt + E)\sin 3t$

8. (8 points) Let $y(t)$ be the solution of the initial value problem

$$y'' - 4y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 0,$$

then $y(1) =$

A. 0

B. e^2

C. $-e^2$

D. $3e^2$

E. $-3e^2$

9. (8 points) If we use the variation of parameters method to find a particular solution y_p of

$$y'' - 2y' + y = t^{-4}e^{5t},$$

we get $y_p(t) = v_1(t)e^t + v_2(t)te^t$. What is $v_1'(t)$?

A. $-t^{-3}e^{5t}$

B. $-t^{-3}e^{4t}$

C. $-t^{-3}e^{6t}$

D. $-t^{-4}e^{5t}$

E. $-t^{-4}e^{4t}$

10. (8 points) Given that e^x is a solution of $xy''(x) - (x+1)y'(x) + y(x) = 0$ for $x > 0$, the function $u(x)e^x$ is another solution if

A. $xu'' + (x-1)u' = 0$

B. $xu'' - (x+1)u' = 0$

C. $xu'' + 2xu' = 0$

D. $u'' - xu' = 0$

E. $u'' + 2xu' = 0$

11. (10 points) Consider a mass-spring system that satisfies $2y''(t) + by'(t) + 50y(t) = 0$. Which of the following is/are true?

(i) If $b = 0$, the motion is critically damped with period $\frac{\pi}{5}$.

(ii) If $b = 12$, the motion is underdamped.

(iii) If $b = 40$, the motion is overdamped.

A. All are true

B. (ii) only

C. (i) and (ii) only

D. (i) and (iii) only

E. (ii) and (iii) only

12. (10 points) Among the following sets, determine which ones are subspaces.

- $S_1 = \{(x, y, z) \in \mathbb{R}^3 : \; x^2 + y^2 + z^2 \leq 1\}$
- $S_2 = \{(5b + 2c, a, c, 3b - c) \in \mathbb{R}^4 : \; a, b, c \in \mathbb{R}\}$
- $S_3 = \{y : [0, 1] \to \mathbb{R} : \; y'' - 4y' + 5y = \sin(t)\}$
- $S_4 = \{p \in \mathbb{P}_3 : \; p(t) = at^3 + (2a - b)t^2 + 5bt - (2a - b) \text{ with } a, b \in \mathbb{R}\}$
- $S_5 = \{y : [0, 1] \to \mathbb{R} : \; y'' - 2y' - 3y = 0\}$

A. S_2, S_4, S_5

B. S_1, S_2, S_5

C. S_2, S_4

D. $S_3, \; S_4$

E. S_4, S_5