April 4, 2023

MA262 - EXAM 2

Green Exam: TEST NUMBER **01**

Instructions

(1) **DO NOT OPEN THIS EXAM BOOKLET UNTIL TOLD TO DO SO**.

(2) Before you open this exam booklet, fill in the information below and use a #2 pencil to fill in the required information on your scantron.

(3) On your scantron, write your 10-digit PUID (starting with "00" from left to right) and your 4-digit Recitation section number (starting with "0" from left to right).

(4) This Green exam is TEST NUMBER **01**.

(5) Once you are allowed to open this booklet, check to make sure you have a complete exam. There are **10** different exam pages including this cover page.

(6) Do any necessary work for each problem in the space provided or on the back of the pages of this exam booklet. No extra paper is allowed. **Circle** your answers in this exam booklet in case of a lost scantron.

(7) There are **11** problems, each worth 9 points and everyone gets 1 point. The maximum possible score is 100. No partial credit will be given.

(8) After you finish your exam, hand in both your scantron and your exam booklet to your instructor, your TA, or one of the proctors.

(9) You may not leave the exam room during the first 20 minutes of the exam. If you do not finish your exam within the first 50 minutes, you must wait until the end of the exam period to leave the room.

Academic Honesty

- Do not seek or obtain any assistance from anyone to answer questions on this exam.
- Do not talk during the exam. Any questions should be directed to your instructor or TA.
- Do not consult notes, books, or calculators during the exam. Do not handle phones, cameras, or any other electronic devices until after you have finished your exam, handed it in to your instructor, your TA or proctor, and left the room.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty may include an **F** in this course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic honesty :

Student Name ______________________ **PUID** ______________

Student Signature ______________________________

Recitation Section # ________ **TA Name** ______________________

1. The matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ has $\det A = 5$. Find the $(1, 2)$ entry of A^{-1}

(the entry in row 1 and column 2 of the matrix A^{-1}).

A. $-\dfrac{2}{5}$

B. $\dfrac{2}{5}$

C. 0

D. $\dfrac{1}{5}$

E. 2

2. Suppose that A and B are 3×3 matrices with $\det(A) = 3$, $\det(B) = 1$. Which of the following statement(s) is/are always **TRUE**?

   (I) The matrix product AB is invertible.

   (II) $\det(2A) = 6$.

   (III) $\det(A^{-1}B) = \frac{1}{3}$.

   A. (I) only

   B. (II) only

   C. (I), (II), and (III)

   D. (I) and (III) only

   E. (II) and (III) only

3. For every 4×2 matrix A, which one of the following statements is always **TRUE** ?

   A. The dimension of the Row Space of A is greater than the dimension of the Column Space of A.

   B. The columns of A must always be *linearly dependent*.

   C. The rows of A must always be *linearly dependent*.

   D. The columns of A must always be *linearly independent*.

   E. The rows of A must always be *linearly independent*.

4. Find all values of the numbers a and b so that the column vector $\mathbf{v} = \begin{bmatrix} 4 \\ a \\ b \end{bmatrix}$ will always belong to the span of the vectors $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$, where

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}.$$

A. All numbers a and b satisfying $b - a = 4$

B. All numbers a and b satisfying $a + b = 5$

C. All numbers a and b satisfying $2a - b = 0$

D. All numbers a and b satisfying $a + b = 4$

E. Only when $a = 8$ and $b = 12$

5. The matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 7 & 9 \\ 0 & 1 & 2 & 3 & 4 \\ -1 & 1 & 3 & 6 & 9 \end{bmatrix}$ has a row-echelon form given by $\operatorname{ref}(A) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Which of the following statement(s) is/are **TRUE**?

$\boxed{\text{I}}$ The set of column vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 1 \\ 0 \end{bmatrix} \right\}$ is a basis for the Column Space of the matrix A.

$\boxed{\text{II}}$ The dimension of the Solution Space to $A\mathbf{x} = \mathbf{0}$ (i.e., the Null Space of A) is 2.

$\boxed{\text{III}}$ The set of row vectors $\left\{ \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 & 2 \end{bmatrix} \right\}$ is a basis for the Row Space of the matrix A.

A. Only $\boxed{\text{I}}$ and $\boxed{\text{III}}$

B. Only $\boxed{\text{I}}$ and $\boxed{\text{II}}$

C. Only $\boxed{\text{II}}$

D. Only $\boxed{\text{III}}$

E. Only $\boxed{\text{II}}$ and $\boxed{\text{III}}$

6. Find all values of k so that the three vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} 0 \\ k \\ 1 \end{bmatrix}$ in $\mathbb{R}^3$ are *linearly independent*.

A. All values of k such that $k \neq -1$

B. $k = -1$

C. All values of k such that $k \neq 1$

D. $k = 1$

E. The vectors are linearly independent for all values of k

7. Which of the following subsets of $\mathbb{R}^3$ are **NOT** subspaces of $\mathbb{R}^3$?

(1) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : \quad x + y + z \leq 0 \right\}$

(2) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : \quad x + 2y + 7z = 2 \right\}$

(3) $\left\{ \begin{bmatrix} 3x \\ -2x \\ 5x \end{bmatrix} : \text{ where } x \text{ are real numbers} \right\}$

(4) $\left\{ \begin{bmatrix} -x \\ 2x \\ 1 \end{bmatrix} : \text{ where } x \text{ are real numbers} \right\}$

A. Only (1) and (3) are not subspaces

B. Only (2) and (4) are not subspaces

C. Only (1), (2) and (3) are not subspaces

D. Only (1), (2) and (4) are not subspaces

E. Only (3) and (4) are not subspaces

8. Find the Wronskian determinant $W(f, g, h)$ for the three functions

$$f(x) = x, \quad g(x) = e^x, \quad h(x) = e^{2x}\,.$$

A. $-e^{3x}$

B. $x(2x+3)e^{3x}$

C. $(2x+3)e^{3x}$

D. $2x-3$

E. $(2x-3)e^{3x}$

9. If $y(x)$ is the solution to the initial value problem $\begin{cases} y'' + 2y' + y = 0 \\ \\ y(0) = 5, \;\; y'(0) = -3 \end{cases}$,

then $y(1) = ?$

A. e^{-1}

B. $2e^{-1}$

C. $5e^{-1}$

D. $7e^{-1}$

E. $9e^{-1}$

10. Determine the general solution to the differential equation

$$y^{(5)} + 4y^{(4)} + 5y''' = 0\,.$$

A. $y = C_1 + C_2\, e^{-2x} \cos x + C_3\, e^{-2x} \sin x$

B. $y = C_1 + C_2\, x + C_3\, x^2 + C_4\, e^{-2x} \cos x + C_5\, e^{-2x} \sin x$

C. $y = C_1 e^{x} + C_2\, x e^{x} + C_3\, x^2 e^{x} + C_4\, e^{-2x} \cos x + C_5\, e^{-2x} \sin x$

D. $y = C_1 + C_2\, x + C_3\, x^2 + C_4\, e^{-x} + C_5\, e^{-5x}$

E. $y = C_1 + C_2\, x + C_3\, x^2 + C_4\, e^{x} \cos 2x + C_5\, e^{x} \sin 2x$

11. A certain spring-mass system leads to this initial value problem:

$$\begin{cases} x'' + 4x = 0 \\ x(0) = 3, \;\; x'(0) = v_0 \end{cases}$$

Find a positive value of v_0 so that the *amplitude* of this spring mass system will be 5.

A. $v_0 = 8$

B. $v_0 = 5$

C. $v_0 = 4$

D. $v_0 = \sqrt{5}$

E. $v_0 = 10$