MA 26200, Fall 2018, Final Exam
Version 1 (Green)

INSTRUCTIONS

(1) **Switch off your phone upon entering the exam room.**
(2) **Do not open the exam booklet until you are instructed to do so.**
(3) **Before you open the booklet, fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
(4) **MARK YOUR TEST NUMBER ON THE SCANTRON**
(5) Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages, including this cover page.
(6) Do any necessary work for each problem on the space provided or on the back of the pages of this booklet. Circle your answers in the booklet.
(7) Use a # 2 pencil to transcribe your answers to the scantron.
(8) After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY

(1) Do not leave the exam during the first 20 minutes of the exam.
(2) Do not leave in the last 10 minutes of the exam.
(3) No talking. Do not seek or obtain any kind of help from anyone to answer the problems on the exam. If you need assistance, consult an instructor.
(4) Do not look at the exam of another student. You may not compare answers with other students until your exam is finished and turned in, and then only after you have left the room.
(5) Your bags must be closed throughout the exam period.
(6) Notes, books, calculators and phones must be in your bags and cannot be used.
(7) Do not handle phones or cameras or any other electronic device until you have finished and turned in your exam, and then only if you have left the room.
(8) When time is called, all students must put down their writing instruments immediately. You must remain in your seat while the TAs will collect the exam booklets and the scantrons.
(9) Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME ______________________________

STUDENT SIGNATURE ______________________________

STUDENT PUID ______________________________

SECTION NUMBER ______________________________

RECITATION INSTRUCTOR ______________________________

1. If $xy' - 3y = x^3$ and $y(1) = 1$, then $y(e) =$

A. e^4
B. e^{-2}
C. $2e^{-3}$
D. $2e^3$
E. e^3

2. The general solution of

$$(2x^2y)y' = -3x^2 - 2xy^2$$

is:

A. $x^2y^3 + y^3 = C$
B. $x^2y^2 + x^3 = C$
C. $x^2y^2 = C$
D. $x^3y^2 + x^2 = C$
E. $x^2y^3 + x = C$

3. The general solution to the equation

$$\frac{dy}{dx} + y = y^2$$

is given by

A. $y(x) = 1 + ce^x$
B. $y(x) = \frac{1}{1+ce^x}$
C. $y(x) = -1 + ce^{-x}$
D. $y(x) = \frac{1}{ce^x-1}$
E. $y(x) = \frac{1}{1-ce^{-x}}$

4. After a suitable change of variable, the equation

$$\frac{dy}{dx} = \frac{y-x}{x}$$

can be transformed into the equation

A. $x\frac{dV}{dx} = -1$

B. $x\frac{dV}{dx} = 1$

C. $x\frac{dV}{dx} = V - 1$

D. $\frac{dV}{dx} = V - 1$

E. $\frac{dV}{dx} = -1$

5. Find an equation involving $a,\ b$ and c so that the system with augmented matrix

$$\begin{bmatrix} 1 & -4 & 7 & a \\ 0 & 3 & -5 & b \\ -2 & 5 & -9 & c \end{bmatrix}$$

is consistent.

A. The system is consistent for **all** values of $a,\ b$ and c.
B. The system is consistent for **no** values of $a,\ b$ and c.
C. The system is consistent when $2a - b + c = 0$.
D. The system is consistent when $2a + b + c = 0$.
E. The system is consistent when $a + b + c = 0$.

6. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}$. What is the $(1, 2)$ entry of $(AB^T)^{-1}$?

A. 0
B. -1
C. 1
D. -2
E. 2

7. Find all values of k so that the vectors $[1,0,2]$, $[0,k,1]$, $[1,1,1]$ are **linearly dependent**.

A. $k=0$
B. $k\neq 1$
C. $k=1$
D. $k\neq -1$
E. $k=-1$

8. Which of the following are vector spaces?

I. All points (x,y) in $\mathbb{R}^2$ satisfying $y\geq 0$.
II. All solutions of $y''-2y'+3y=0$ on $(-\infty,\infty)$.
III. All solutions of $x^2y'''+(\cos x)y''+xy=0$ on $(0,\infty)$.
IV. All column vectors $\mathbf{x}=\begin{bmatrix}x_1\\x_2\end{bmatrix}$ in $\mathbb{R}^2$ satisfying $\begin{bmatrix}1&2\\3&4\end{bmatrix}\mathbf{x}=3\mathbf{x}$.

A. IV only
B. I and IV only
C. II and III only
D. II and IV only
E. II, III and IV only

9. If A and B are 4×4 matrices with $\det A = 2$ and $\det B = 6$, what is $\det(3AB^{-1})$?

A. 27
B. 3
C. 18
D. 9
E. 1

10. Find a basis for the space spanned by the vectors

$$\begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \begin{bmatrix}-2\\1\\-1\\1\end{bmatrix}, \begin{bmatrix}6\\-1\\2\\-1\end{bmatrix}, \begin{bmatrix}5\\-3\\3\\-4\end{bmatrix}.$$

A. $\begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \begin{bmatrix}-2\\1\\-1\\1\end{bmatrix}, \begin{bmatrix}6\\-1\\2\\-1\end{bmatrix}, \begin{bmatrix}5\\-3\\3\\-4\end{bmatrix}.$

B. $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}-2\\1\\0\\0\end{bmatrix}, \begin{bmatrix}6\\-1\\1\\0\end{bmatrix}.$

C. $\begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \begin{bmatrix}-2\\1\\-1\\1\end{bmatrix}, \begin{bmatrix}6\\-1\\2\\-1\end{bmatrix}.$

D. $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}-2\\1\\0\\0\end{bmatrix}.$

E. $\begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \begin{bmatrix}-2\\1\\-1\\1\end{bmatrix}.$

11. If the dimension of the nullspace of a 5×6 matrix A is 4, what are the rank of A and the dimension of the column space of A?

A. rank(A)=3, dim[Col(A)]=3.
B. rank(A)=1, dim[Col(A)]=2.
C. rank(A)=2, dim[Col(A)]=1.
D. rank(A)=1, dim[Col(A)]=1.
E. rank(A)=2, dim[Col(A)]=2.

12. Which of the following statements are correct?
I. A square matrix A is not invertible if and only if $\lambda = 0$ is an eigenvalue of A.
II. If $\mathbf{v_1}$ and $\mathbf{v_2}$ are linearly independent eigenvectors of a matrix A, then they correspond to different eigenvalues.
III. If a square matrix A is triangular, then the eigenvalues of A are the entries on its main diagonal.

A. I and III only.
B. II and III only.
C. I and II only.
D. I only.
E. III only.

13. Let $A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 4 & 0 \end{bmatrix}$. Then the sum of the three eigenvalues of A is

A. 0
B. 1
C. 2
D. 3
E. 4

14. Find the solution to the initial value problem

$$y'' - 3y' + 2y = 0; \quad y(0) = 0, \quad y'(0) = 1.$$

A. $1 - e^t$
B. $e^t - e^{2t}$
C. $-e^t + e^{2t}$
D. $-te^t + te^{2t}$
E. The problem has no solution

15. Let $y_p(t)$ be a particular solution of $y'' - y' = 6e^t + 2e^{2t}$. Then $y_p(t) =$

A. $3te^t + \frac{1}{2}e^{2t}$
B. $6te^t + e^{2t}$
C. $12te^t + 2e^{2t}$
D. $12e^t + 2e^{2t}$
E. $6e^t + e^{2t}$

16. Which form should you use to determine a particular solution of the differential equation

$$y'' - 6y' + 25y = t^2 + e^{3t}\cos 4t?$$

A. $A_1t^2 + A_2t + A_3 + A_4e^{3t}\cos 4t + A_5e^{3t}\sin 4t$
B. $A_1t^2 + A_2t^2e^{3t}\cos 4t + A_3t^2e^{3t}\sin 4t$
C. $A_1t^2 + A_2e^{3t}\cos 4t + A_3e^{3t}\sin 4t$
D. $A_1t^2 + A_2t + A_3 + A_4te^{3t}\cos 4t + A_5te^{3t}\sin 4t$
E. $A_1t^2 + A_2e^{3t}\cos 4t + A_3te^{3t}\cos 4t + A_4e^{3t}\sin 4t + A_5te^{3t}\sin 4t$

17. The functions $y(t) = t^2$ and $y(t) = t^3$ are solutions of the homogeneous differential equation

$$t^2 y'' - 4ty' + 6y = 0 \quad \text{on } (0, \infty).$$

Using variation of parameters with $y_p(t) = v_1(t)t^2 + v_2(t)t^3$ to find a solution of the nonhomogeneous differential equation $t^2y'' - 4ty' + 6y = 4t^3$, what do you get for $v_1'(t)$?

A. 4
B. -4
C. $4t^2$
D. $-4t^2$
E. $-4t$

18. On what interval I does the existence-uniqueness theorem apply to give a unique solution of

$$t(t-4)y'' + 2ty' - y = \sin t, \quad y(3) = 1, \quad y'(3) = -6$$

on I?

A. $(-\infty, \infty)$
B. $(0, \infty)$
C. $(0, 4)$
D. $(-\infty, 4)$
E. $(-\infty, 0)$

19. The motion of a mass-spring system is governed by

$$y'' + by' + 16y = 0, \qquad y(0) = 1,\ y'(0) = -4.$$

Which of the following statements are correct?
I. When $b = 0$, the motion is undamped, with amplitude $A = \sqrt{2}$ and phase angle $\phi = -\pi/4$.
II. When $b = 0$, the motion is undamped, with amplitude $A = 1$ and phase angle $\phi = \pi/4$.
III. When $b = 8$, the motion is underdamped, with damping factor e^{-4t}.
IV. When $b = 10$, the motion is overdamped, and $y(t) \to 0$ as $t \to \infty$.

A. I and IV only.
B. II and IV only.
C. I, III, and IV only.
D. II and III only.
E. II, III and IV only.

20. The general solution to the equation

$$(D-1)^2(D+3)(D^2+2D+5)^2[y] = 0$$

is
A. $y(x) = C_1e^x + C_2e^{-x} + C_3e^{-3x} + C_4e^{-x}\cos 2x + C_5e^{-x}\sin 2x + C_6xe^{-x}\cos 2x + C_7xe^{-x}\sin 2x$
B. $y(x) = C_1e^x + C_2xe^x + C_3e^{-3x} + C_4\cos 2x + C_5\sin 2x + C_6x\cos 2x + C_7x\sin 2x$
C. $y(x) = C_1e^x + C_2xe^x + C_3e^{-3x} + C_4e^{-x}\cos 2x + C_5e^{-x}\sin 2x + C_6xe^{-x}\cos 2x + C_7xe^{-x}\sin 2x$
D. $y(x) = C_1e^x + C_2xe^x + C_3x^2e^x + C_4e^{-3x} + C_5xe^{-3x} + C_6e^{-x}\cos 2x + C_7e^{-x}\sin 2x$
E. $y(x) = C_1e^x + C_2xe^x + C_3e^{-3x} + C_4xe^{-x}\cos 2x + C_5xe^{-x}\sin 2x + C_6x^2e^{-x}\cos 2x + C_7x^2e^{-x}\sin 2x$

21. Which one of the following is an annihilator for the function $f(x) = e^x + x^2$?

A. D^3
B. $(D-1) + D^2$
C. $(D-1) + D^3$
D. $(D-1)D^2$
E. $(D-1)D^3$

22. Consider the following sets of vector functions:

(i) $S_1 = \left\{ \begin{bmatrix} 1 \\ |t| \end{bmatrix}, \begin{bmatrix} |t| \\ t^2 \end{bmatrix} \right\}$;

(ii) $S_2 = \{\mathbf{x}_1, \mathbf{x}_2\}$, where $\mathbf{x}_1 = \begin{bmatrix} e^t \\ -2e^t \end{bmatrix}$ and $\mathbf{x}_2 = \begin{bmatrix} -2e^t \\ 4e^t \end{bmatrix}$ are solutions to a system of differential equations $\mathbf{x}'(t) = A\mathbf{x}(t)$ with a 2×2 real constant coefficient matrix A;

(iii) $S_3 = \{\mathbf{x}_1, \mathbf{x}_2\}$, where $\mathbf{x}_1 = \begin{bmatrix} e^t \\ 2e^t \end{bmatrix}$ and $\mathbf{x}_2 = \begin{bmatrix} 2e^{2t} \\ e^{2t} \end{bmatrix}$ are solutions to a system of differential equations $\mathbf{x}'(t) = A\mathbf{x}(t)$ with a 2×2 real constant coefficient matrix A.

Which of the sets are **linearly independent** on $(-\infty, \infty)$?

A. S_1 and S_2 only
B. S_3 only
C. S_1 and S_3 only
D. S_1, S_2, and S_3
E. None of the sets

23. Let $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ be the solution to the initial value problem

$$\mathbf{x}'(t) = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} \mathbf{x}(t), \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Then $x_2(t) =$

A. 1
B. e^{2t}
C. e^t
D. $2e^{2t} - e^t$
E. $-e^{2t}$

24. Let A be a 2×2 real constant coefficient matrix. Suppose the system of differential equations $\mathbf{x}'(t) = A\mathbf{x}(t)$ has a fundamental matrix $X(t) = \begin{bmatrix} 2e^t & e^{2t} \\ e^t & e^{2t} \end{bmatrix}$. When the method of variation of parameters is used to find a particular solution of the form
$\mathbf{x}_p(t) = X(t) \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$ to $\mathbf{x}'(t) = A\mathbf{x}(t) + \begin{bmatrix} e^t \\ -e^t \end{bmatrix}$, which of the following is a correct choice for $v_1(t)$?

A. $2t$
B. t^2
C. e^t
D. $3e^{-t}$
E. $2e^{2t}$