MA 262 Spring 2001
FINAL EXAM INSTRUCTIONS

NAME ______________________ INSTRUCTOR ______________________

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. On the mark–sense sheet, fill in the instructor's name and the course number.

3. Fill in your name and student identification number and blacken in the appropriate spaces.

4. Mark in the section number, the division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number ask your instructor.)

5. Sign the mark–sense sheet.

6. Fill in your name and your instructor's name above and on the first page of the question sheets.

7. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.

8. No partial credit will be given, but if you show your work on the question sheets it may be considered if your grade is on the borderline.

9. NO CALCULATORS, BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

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1. If $y = y(x)$ is a solution to the equation $\frac{dy}{dx} = \sqrt{y}(4x+2)$, $\quad y(1) = 1$, then $y(2) =$

A. 16

B. 36

C. 25

D. $\frac{9}{2}$

E. $\frac{15}{2}$

2. The solution of $\frac{dy}{dx} = \frac{x^2}{3y^2} + \frac{y}{x}$ that satisfies $y(1) = 2$ is

A. $y^3 = \ln x + 2$

B. $y = x(\ln x)^{\frac{1}{3}} + 2$

C. $y^3 = x^3 \ln x + 2$

D. $y^3 = \ln x + 8$

E. $y^3 = x^3 \ln x + 8x^3$

3. Find the general solution of $2xydx + (x^2+1)dy = 0$.

A. $x^2y^2 + x = c$

B. $y = e^{\tan^{-1} x} + c$

C. $x^2y + 1 = c$

D. $x^2y + y = c$

E. $\ln y = -2x \tan^{-1} x + c$

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4. Two minutes after a hot metal object is taken from a furnace, the temperature of the object is 320°F. One minute later its temperature is 240°F. By Newton's law of cooling, the temperature of the object $T(t)$ satisfies a differential equation

$$\frac{dT(t)}{dt} = -k(T(t) - T_m), \qquad T(0) = T_0.$$

If the temperature of the room is 80°F, what was the temperature of the object when it was removed from the furnace?

A. 540°F

B. 620°F

C. 640°F

D. $\frac{1280}{3}$°F

E. $\frac{5120}{9}$°F

5. Suppose that a salt solution with a concentration of 1 lb/gal is poured into a tank at a rate of 2 gal/min., and the well-stirred solution flows out of the tank also at a rate of 2 gal/min. If the tank initially contains 4 gallons of solution with a concentration of 2 lb/gal, what is the <u>concentration</u> of the solution in the tank after t minutes?

A. $\frac{3}{2} + \frac{1}{2}e^{-t}$

B. $2 - e^{-t/2}$

C. $2e^{-t/2}$

D. $1 + e^{-t/2}$

E. $3 - e^{-2t}$

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6. If $M = \begin{bmatrix} 3 & 5 & -12 \\ 2 & 3 & -7 \\ -2 & -1 & 1 \end{bmatrix}$. Then the rank of M is

A. 0

B. 1

C. 2

D. 3

E. 4

7. If the 2×2 matrices $M = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$. Then the second row of M^{-1} is

A. $\left(\frac{1}{2}, -\frac{1}{4}\right)$

B. $\left(-\frac{1}{2}, \frac{3}{4}\right)$

C. $\left(\frac{1}{2}, -\frac{1}{2}\right)$

D. $\left(-\frac{1}{4}, \frac{3}{4}\right)$

E. $\left(\frac{1}{2}, -\frac{3}{4}\right)$

8. If A and B are 3×3 matrices such that $\det(A) = 2$, $\det(B) = 3$, then $\det(2A^{-1}B^2)$ equals

A. 36

B. $\frac{1}{9}$

C. 9

D. $\frac{1}{36}$

E. Undefined

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9. If $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 7 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$, and denote $A^{-1} = B = (b_{ij})$. Find b_{23}.

A. $-\frac{1}{2}$

B. $-\frac{3}{10}$

C. 0

D. $\frac{3}{10}$

E. $\frac{1}{2}$

10. Let $T : \mathbb{R}^4 \mapsto \mathbb{R}^3$ be given by $T(x) = Ax$, where

$$A = \begin{bmatrix} 1 & 0 & 2 & 9 \\ 1 & 1 & 3 & 13 \\ -1 & 1 & -1 & -5 \end{bmatrix}.$$

What is the dimension of Rng(T)?

A. 1

B. 2

C. 3

D. 4

E. 0

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11. Which of the following is true?

i) $S = \{y \in C^2(I) : y'' + y' - 5y = 1\}$ is a subspace of $C^2(I)$.

ii) $U = \{(0,0,0), (0,1,0)\}$ is a subspace of $\mathbb{R}^3$.

iii) $V = \{(x,y) \in \mathbb{R}^2 : x + y = 0\}$ is a subspace of $\mathbb{R}^2$.

A. ii) only

B. i) and iii)

C. ii) and iii)

D. iii) only

E. None of i)–iii)

12. Which of the following sets of vectors form a basis for $\mathbb{R}^3$?

A. $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\2\\4\end{bmatrix}, \begin{bmatrix}1\\3\\5\end{bmatrix}$

B. $\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$

C. $\begin{bmatrix}2\\1\\0\end{bmatrix}, \begin{bmatrix}1\\2\\0\end{bmatrix}$

D. $\begin{bmatrix}0\\1\\-1\end{bmatrix}, \begin{bmatrix}4\\3\\2\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}-1\\0\\0\end{bmatrix}$

E. $\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\end{bmatrix}, \begin{bmatrix}0\\0\\6\end{bmatrix}$

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13. Find a such that $\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ is in the span$\left\{ \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 5 & 3 \\ 2 & a \end{bmatrix} \right\}$?

A. $a = 1$

B. $a = 2$

C. $a = 0$

D. $a = -2$

E. $a = -1$

14. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(1,2) = (1,3)$ and $T(3,2) = (-3,5)$, then $T(1,0) =$

A. $(-2,1)$

B. $(-4,2)$

C. $(-2,8)$

D. $(-1,4)$

E. $(4,2)$

15. Let $AA^T = \begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}$ where $A = \begin{bmatrix} 1 & y \\ x & -1 \end{bmatrix}$. Then $(x,y) =$

A. $(1,1)$

B. $(2,1)$

C. $(1,5)$

D. $(2,5)$

E. $(1,2)$

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16. An annilhilator of $\cos x + xe^x + 1$ is

A. $D(D-1)(D+1)^2$
B. $(D^2-1)(D+1)^2$
C. $D(D-1)(D^2+1)^2$
D. $D(D-1)^2(D^2+1)$
E. $D(D-1)(D^2+1)$

17. The general solution of $y'' - 3y' - 4y = 0$ is $y = C_1e^{-x} + C_2e^{4x}$. Which of the following (for suitable constants) is a particular solution of the differential equation $y'' - 3y' - 4y = xe^{-x} + \cos 2x$?

A. $Axe^{-x} + B\cos 2x$
B. $Axe^{-x} + B\cos 2x + C\sin 2x$
C. $x(A+Bx)e^{-x} + C\cos 2x$
D. $x(A+Bx)e^{-x} + C\cos 2x + D\sin 2x$
E. $Ae^{-x} + B\cos 2x + C\sin 2x$

18. Using the method of variation of parameters, we know that a particular solution to the differential equation $y'' + y' - 6y = -5\ln(1+x^2)$ is of the form $u_1e^{2x} + u_2e^{-3x}$. What is u_2'?

A. $e^{3x}\ln(1+x^2)$
B. $-5e^{2x}\ln(1+x^2)$
C. $-e^{-2x}\ln(1+x^2)$
D. $-5e^{-3x}\ln(1+x^2)$
E. $e^{-2x}\ln(1+x^2)$

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19. Given that $y_1 = e^{-x}$ is a solution to the differential equation

$$xy'' + (1+2x)y' + (x+1)y = 0.$$

Suppose that $y_2 = ue^{-x}$ is another solution, then u satisfies the differential equation

A. $u'' - 2u' + u = 0$

B. $xu''e^{-x} - 2xu' = 0$

C. $u' - u = 0$

D. $xu'' + u' = 0$

E. $xu'' + (1+2x)u' + (x+1)u = 0$

20. Let y_1 and y_2 be solutions of the nonhomogeneous differential equation

$$y'' + a_1(x)y' + a_2(x)y = F(x),$$

then which of the following functions is also a solution of the same differetial equation?

A. $y_1 - y_2$

B. $y_1 + y_2$

C. $2y_1 - y_2$

D. $y_1 - 2y_2$

E. All of the above

21. Let $y(x)$ be the solution to the initial value problem

$$y'' + 3y' - 4y = 6e^{2x}, \qquad y(0) = 2,\ y'(0) = 3.$$

What is $y(1)$?

A. $-e + 2e^{-4} + e^2$

B. $e^{-4} + e^2$

C. $2e - e^{-4} + e^2$

D. $e + e^2$

E. $e + e^{-4}$

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22. One eigenvalue of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ -1 & 4 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ is $\lambda = 3$. A basis of the corresponding eigenspace is

A. $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

E. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

23. Which of the following matrices is defective

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}.$$

A. A and B

B. A and C

C. B only

D. C only

E. A only

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24. The general solution of the system of first order differential equations

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

is:

A. $C_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} e^{-3t} + C_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} e^{3t} + C_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{-t}$

B. $C_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} te^{3t} + C_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{-t}$

C. $C_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} e^{-3t} + C_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} e^{-3t} + C_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{t}$

D. $C_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} e^{3t} + C_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{t}$

E. $C_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} e^{3t} + C_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{-t}$

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25. If a fundamental matrix for $\mathbf{x}' = A\mathbf{x}$ is $X(t) = \begin{bmatrix} e^{-2t} & e^{2t} \\ 0 & 2e^{2t} \end{bmatrix}$, then a particular solution to the nonhomogeneous system $\mathbf{x}' = A\mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is

A. $\begin{bmatrix} e^{-2t} & e^{2t} \\ 0 & 2e^{2t} \end{bmatrix} \begin{bmatrix} e^{2t} \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} e^{-2t} & e^{2t} \\ 0 & 2e^{2t} \end{bmatrix} \begin{bmatrix} \frac{1}{2}e^{2t} \\ 0 \end{bmatrix}$

C. $\begin{bmatrix} e^{2t} & -\frac{1}{2}e^{-2t} \\ 0 & \frac{1}{2}e^{-2t} \end{bmatrix} \begin{bmatrix} e^{2t} \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} e^{2t} & -\frac{1}{2}e^{-2t} \\ 0 & \frac{1}{2}e^{-2t} \end{bmatrix} \begin{bmatrix} \frac{1}{2}e^{2t} \\ 0 \end{bmatrix}$

E. None of the above