MA262 — FINAL EXAM — SPRING 2016 — MAY 2, 2016
TEST NUMBER 01

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 12 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
6. Each problem is worth 10 points. The maximum possible score is 200 points. No partial credit.
7. Use a # 2 pencil to fill in the answers on your scantron
8. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY:

1. Do not leave the exam room during the first 20 minutes of the exam.
2. If you do not finish your exam in the first 110 minutes, you must wait until the end of the exam period to leave the room.
3. Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
4. Do not look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes, books, calculators.
6. Do not handle phones or cameras, or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ____________________

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SECTION NUMBER ____________________

RECITATION INSTRUCTOR: ____________________

1. Which of the following functions is an integrating factor for the differential equation

$$\frac{dy}{dx} = (\ln x)y + e^x?$$

A. $e^{x \ln x}$

B. $e^{x \ln x - x}$

C. $e^{x - x \ln x}$

D. $e^{\frac{1}{x}}$

E. x

2. Consider the differential equation

$$\frac{dP}{dt} = 5\left(1 - \frac{P}{4}\right) P.$$

Which of the following statements are true?

I. The equilibrium solutions are $P = 0$ and $P = 4$.

II. The solution $P(t)$ with $P(0) = 2$ converges to the solution $P = 4$ as $t \to +\infty$.

III. The solution $P(t)$ with $P(0) = 2$ converges to the solution $P = 0$ as $t \to +\infty$.

IV. The solution $P(t)$ with $P(0) = 5$ converges to the solution $P = 4$ as $t \to +\infty$.

A. (I) and (IV) only
B. (I) and (II) only
C. (I), (II), and (III) only
D. (I), (II) and (IV) only
E. (II) and (IV) only

3. Let $y(x)$ be the solution to the initial value problem

$$\frac{dy}{dx} = \frac{y}{x(\ln x)^2} \qquad \text{and} \qquad y(e) = e.$$

Then $\ln y(e^2)$ is equal to

A. 2

B. 3

C. $\dfrac{3}{4}$

D. $\dfrac{3}{2}$

E. 1

4. Let $y(x)$ satisfy the following initial value problem

$$(3x^2e^{xy} + x^3ye^{xy})\,dx + (x^4e^{xy})\,dy = 0, \quad y(1) = 0.$$

Then $y(2)$ is equal to

A. $\ln 2$

B. $-\ln 2$

C. $\dfrac{3}{4}\ln 2$

D. $-\dfrac{2}{3}\ln 2$

E. $-\dfrac{3}{2}\ln 2$

5. Let $y(x)$ satisfy the following initial value problem

$$y'' + y^{-1}(y')^2 = y^{-1}e^{-y}y', \quad y(0) = 1, \; y'(0) = 1.$$

Then $V(x) = y'(x)$ satisfies

A. $V(x) = y^{-1}(1 + \frac{1}{e} - e^{-y})$

B. $V(x) = \frac{1}{2}y^{-1}(2 + \frac{1}{e} - e^{-y})$

C. $V(x) = -y^{-1}e^{-y} + \frac{1}{e} + 1$

D. $V(x) = y^{-1}(1 - \frac{1}{e} + e^{-y})$

E. $V(x) = \frac{3}{2}y^{-1}(\frac{2}{3} - \frac{1}{e} + e^{-y})$

6. The rank of $\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & 3 & 1 \\ 3 & 7 & 6 & 13 \\ 2 & 5 & 5 & 9 \end{bmatrix}$ is equal to

A. 3

B. 4

C. 1

D. 0

E. 2

7. Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^5$ be the Linear transformation given by $T(x) = Ax$, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & -1 & 1 \\ 0 & -3 & -2 \\ 1 & 1 & 1 \end{bmatrix}$.

Then the dimension of the range of T is equal to

A. 1

B. 2

C. 3

D. 4

E. 5

8. Let $A = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 5 & -1 \\ 1 & 6 & -2 \end{bmatrix}$. Let λ_1, λ_2 and λ_3 denote the eigenvalues of A and let E_1, E_2 and E_3 denote the corresponding eigenspaces. Which of the following is correct?

A. $\lambda_1 = 2$, $\lambda_2 = 3$ and $\lambda_3 = -1$, $\dim E_1 = \dim E_2 = \dim E_3 = 1$

B. $\lambda_1 = \lambda_2 = -1$, $\lambda_3 = 4$, $\dim E_1 = \dim E_2 = 2$, $\dim E_3 = 1$

C. $\lambda_1 = \lambda_2 = -1$, $\lambda_3 = 4$, $\dim E_1 = \dim E_2 = 1$, $\dim E_3 = 1$

D. $\lambda_1 = 1$, $\lambda_2 = \lambda_3 = 4$, $\dim E_1 = 1$, $\dim E_2 = \dim E_3 = 2$

E. $\lambda_1 = \lambda_2 = -1$, $\lambda_3 = 3$, $\dim E_1 = \dim E_2 = 2$, $\dim E_3 = 1$

9. Find all values of α such that the vectors $(1, 0, \alpha/4, 1)$, $(2, 0, 1, \alpha)$ and $(1, 0, 1, 2)$ are linearly dependent

A. $\alpha = 1$ and $\alpha = 2$

B. $\alpha = 2$ and $\alpha = 6$

C. $\alpha = 2$ and $\alpha = 4$

D. $\alpha = -2$ and $\alpha = 4$

E. $\alpha = 3$ and $\alpha = 6$

10. Let A and B be square matrices such that $\det(A) = 5$ and $\det(A + B) = 20$. We conclude that $\det(I + A^{-1}B)$ is equal to

A. 4

B. 3

C. 5

D. 10

E. 8

11. Let $M_3(\mathbb{R})$ be the space of 3×3 matrices with real entries and let S be the subspace of $M_3(\mathbb{R})$ such that the sums of the elements of each row is equal to zero. The dimension of S is equal to

A. 3

B. 4

C. 1

D. 2

E. 6

12. A particular solution to the equation

$$(D^2 - 2D + 10)^2 y = e^x \sin 3x + \cos 2x,$$

(which does not contain terms that solve the homogeneous equation) has the form

A. $y_p(x) = e^x(A_1 \cos 3x + B_1 \sin 3x + A_2 x \cos 3x + B_2 x \sin 3x + A_3 x^2 \cos 3x + B_3 x^2 \sin 3x) + A_5 \cos 2x + B_5 \sin 2x$

B. $y_p(x) = e^x(A_1 x \cos 3x + B_1 x \sin 3x + A_2 x^2 \cos 3x + B_2 x^2 \sin 3x) + A_3 \cos 2x + B_3 \sin 2x$

C. $y_p(x) = e^x(A_1 \cos 3x + B_1 \sin 3x) + A_2 \cos 2x + B_2 \sin 2x$

D. $y_p(x) = e^x(A_1 x^2 \cos 3x + B_1 x^2 \sin 3x) + A_2 \cos 2x + B_2 \sin 2x$

E. $y_p(x) = e^x(A_1 x \cos 3x + B_1 x \sin 3x) + A_2 \cos 2x + B_2 \sin 2x$

13. Find the general solution to the differential equation $y^{(4)} - 8y'' + 16y = 0$.

A. $y = c_1 e^{2x} + c_2 e^{-2x}$

B. $y = c_1 x e^{2x} + c_2 x e^{-2x}$

C. $y = c_1 e^{2x} + c_2 e^{-2x} + c_3 x e^{2x} + c_4 x e^{-2x}$

D. $y = c_1 x e^{2x} + c_2 x e^{-2x} + c_3 x^2 e^{2x} + c_4 x^2 e^{-2x}$

E. $y = c_1 \cos 2x + c_2 \sin 2x + c_3 x \cos 2x + c_4 x \sin 2x$

14. Which of the following is a Green's function of the differential equation $y'' - 3y' + 2y = F(x)$? $\left(K(x,t) = \frac{1}{W[y_1,y_2](t)}\left(y_1(t)y_2(x) - y_2(t)y_1(x)\right)\right)$

A. $K(x,t) = (e^{2x+4t} - e^{x+5t})$

B. $K(x,t) = (e^{2(x-t)} - e^{(x-t)})$

C. $K(x,t) = (e^{3(x+t)} - e^{x+t})$

D. $K(x,t) = (e^{4(x-t)} - e^{5(x-t)})$

E. $K(x,t) = (e^{3(x-t)} + e^{2(x-t)})$

15. It is given that $y_p = -\frac{1}{2}x\cos x$ is a particular solution to the differential equation $y'' + y = \sin x$. Let y be the solution to the initial value problem

$$y'' + y = \sin x,\ \ y(0) = 0, y'(0) = 1.$$

Find the value of $y(\frac{\pi}{2})$.

A. $\dfrac{\pi}{4}$

B. 3π

C. $\dfrac{3}{2}$

D. $\dfrac{1}{2}$

E. 1

16. Find all the values of a and b such that the system of equations $\begin{cases} x_1 & & +2x_3 & = 1 \\ x_1 & +3x_2 & +x_3 & = 0 \\ x_1 & & +ax_3 & = b \end{cases}$ has infinitely many solution.

A. $a = 1, b = 2$

B. $a \neq 2, b \neq 1$

C. $a = 2, b \neq 1$

D. $a \neq 1, b \neq 2$

E. $a = 2, b = 1$

17. For the inverse of the matrix $\begin{bmatrix} 0 & 3 & 0 \\ 2 & 1 & 1 \\ 4 & 4 & 1 \end{bmatrix}$, the entry in the third row and second column is

A. 2

B. -1

C. 3

D. -6

E. 4

18. If $y_1 = t^4$ is a solution of the differential equation $t^2y'' - 7ty' + 16y = 0$ for $t > 0$, and $y_2 = y_1 v$ is another solution of the same differential equation, then v satisfies which of the following equation?

A. $t^2v'' - 7tv' + 16v = 0$

B. $tv'' + v' = 0$

C. $tv' + 4v = 0$

D. $tv'' - v' = 0$

E. $tv' - 4v = 0$

19. Let $A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$. The eigenvalues of A are $\lambda_1 = 5$ and $\lambda_2 = -5$, and corresponding eigenvectors are $V_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $V_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. Let $\mathbf{x}(t)$ be the solution to the initial value problem

$$\mathbf{x}'(t) = A\mathbf{x}(t), \quad \mathbf{x}(0) = \begin{bmatrix} 4 \\ -3 \end{bmatrix}.$$

Then $\mathbf{x}(\frac{1}{5}\ln 2)$ is equal to

A. $\mathbf{x}(\frac{1}{5}\ln 2) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

B. $\mathbf{x}(\frac{1}{5}\ln 2) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

C. $\mathbf{x}(\frac{1}{5}\ln 2) = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$

D. $\mathbf{x}(\frac{1}{5}\ln 2) = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$

E. $\mathbf{x}(\frac{1}{5}\ln 2) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

20. Find a particular solution of the following system of nonhomogeneous differential equations

$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{-2t}.$$

of the form $\mathbf{x}_p(t) = e^{-2t} \begin{bmatrix} a \\ b \end{bmatrix}$.

A. $\mathbf{x}_p(t) = e^{-2t} \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

B. $\mathbf{x}_p(t) = e^{-2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

C. $\mathbf{x}_p(t) = e^{-2t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

D. $\mathbf{x}_p(t) = e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

E. $\mathbf{x}_p(t) = e^{-2t} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$