Spring 2017

# MA 262 - FINAL EXAM

**GREEN - Test Version 01**

## Instructions

1. Fill in your scantron with your `NAME, PUID, Section Number` (4 digits), and the correct `Test Version` (GREEN is 01).
2. This exam contains 25 problems worth 8 points each, for a total of 200 points.
3. Do all your work only in the spaces provided or on the backside of the pages. Show your work.
4. Mark your answers clearly on your scantron. In addition, also **CIRCLE** your answer choice for each problem in this exam booklet in case your scantron is lost.
5. NO books, notes, calculators, phones, or cameras are allowed on this exam. Turn off and put away all electronic devices.

## Academic Dishonesty

- *Students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.*
- *You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.*
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**I have read and understand the above statements regarding academic dishonesty:**

**STUDENT NAME** ____________________

**STUDENT PUID #** ____________________

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**SECTION #**__________ **Recitation INSTRUCTOR** __________

| Sect # | Time | Recitation Instructor |
|---|---|---|
| 0153 | 8:30am | Sokurski |
| 0171 | 9:30am | Sokurski |
| 0260 | 9:30am | Cooper |
| 0284 | 10:30am | Cooper |
| 0306 | 4:30pm | Zhang |
| 0339 | 3:30pm | Zhang |
| 0420 | 11:30am | Ponce |
| 0464 | 12:30pm | Ponce |

| Sect # | Time | Recitation Instructor |
|---|---|---|
| 0525 | 3:30pm | Parab |
| 0570 | 4:30pm | Parab |
| 0612 | 2:30pm | Liu |
| 0636 | 1:30pm | Liu |
| 0707 | 1:30pm | Solapurkar |
| 0721 | 2:30pm | Solapurkar |

**1.** The general solution to $xy' - y = \dfrac{x^2}{e^x}$ for $x > 0$ is

**A.** $y = -e^{-x} + c$

**B.** $y = -xe^{-x} + cx$

**C.** $y = -cxe^{-x}$

**D.** $y = -ce^{-x} + x$

**E.** $y = -cxe^{-x} - x$

**2.** Let $y(x)$ be the solution to the initial value problem

$$(x^2 + 1)\,\frac{dy}{dx} = -2xy, \qquad y(1) = 2.$$

Then $y(0) =$

**A.** 4

**B.** 3

**C.** 2

**D.** 1

**E.** 0

**3.** Which of the following is the solution to the initial value problem

$$(e^x \sin y - 2y \sin x - 1) + (e^x \cos y + 2\cos x + 3)\,\frac{dy}{dx} = 0, \quad y(0) = \pi\,?$$

**A.** $e^x \sin y + 2y\cos x + 3x - y = \pi$

**B.** $e^x \cos y - 2y\cos x - x + 3y = \pi$

**C.** $e^x \cos y + 2\sin x + 3x - y = -1 - \pi$

**D.** $e^x \sin y + 2y\cos x + 3y - x = 5\pi$

**E.** $e^x \sin y - 2y\cos x - x + 3y = \pi$

**4.** Initially a 100-gallon tank is **half full** of pure water. A salt solution containing 0.5 lb of salt per gallon runs into the tank at a rate of 4 gal/min. The well mixed solution runs out of the tank at a rate of 3 gal/min. Let $A(t)$ be the amount of salt in the tank at time $t$. Which of the following initial value problems describes $A(t)$ before the tank becomes full?

**A.** $\dfrac{dA}{dt} = 2 - \dfrac{3A}{t+50}, \quad A(0) = 0$

**B.** $\dfrac{dA}{dt} = 2 - \dfrac{3A}{2t+50}, \quad A(0) = 0$

**C.** $\dfrac{dA}{dt} = 3 - \dfrac{A}{t+50}, \quad A(0) = 100$

**D.** $\dfrac{dA}{dt} = 3 - \dfrac{3A}{50-t}, \quad A(0) = 50$

**E.** $\dfrac{dA}{dt} = 2 - \dfrac{3A}{t+100}, \quad A(0) = 0$

**5.** Consider the equation

$$\frac{dy}{dx} + \frac{1}{4x}\,y = xy^3.$$

If $v = y^{-2}$, then $v$ satisfies

**A.** $\dfrac{dv}{dx} - \dfrac{1}{4x}\,v = -2x$

**B.** $\dfrac{dv}{dx} + \dfrac{1}{x}\,v = -x$

**C.** $\dfrac{dv}{dx} + \dfrac{1}{2x}\,v = x$

**D.** $\dfrac{dv}{dx} - \dfrac{1}{x}\,v = 2x$

**E.** $\dfrac{dv}{dx} - \dfrac{1}{2x}\,v = -2x$

**6.** Find the general solution to

$$y'' + \frac{1}{x}\,y' = 3x, \quad x > 0.$$

**A.** $y = \frac{1}{3}x^3 + c_1 x + c_2$

**B.** $y = x^3 + \dfrac{c_1}{x} + c_2$

**C.** $y = x^2 + \dfrac{c_1}{x} + c_2$

**D.** $y = \frac{1}{3}x^3 + c_1 \ln x + c_2$

**E.** $y = x^3 + c_1 \ln x + c_2$

**7.** The sum of the diagonal elements of the inverse of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$,

**A.** $\frac{5}{2}$

**B.** $-1$

**C.** $-\frac{1}{2}$

**D.** $0$

**E.** $3$

**8.** If $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$, solve the system $A^T(A\,\mathbf{x}) = A^T\mathbf{b}$.

**A.** $\mathbf{x} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

**B.** $\mathbf{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

**C.** $\mathbf{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

**D.** $\mathbf{x} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

**E.** No solution since $A\mathbf{x} = \mathbf{b}$ is inconsistent

**9.** Let $A, B$, and $C$ be $3 \times 3$ matrices with $\det B = 1$ and $\det C = 3$. If $\det(-2A^{-1}BC^2) = 6$, then $\det A = ?$

**A.** $9$

**B.** $-8$

**C.** $-12$

**D.** $-9$

**E.** $-3$

**10.** If $A$ is a $3 \times 5$ matrix with $\mathsf{rank}(A) = 3$, which one of the following statements must be **TRUE**?

**A.** $\mathsf{rank}\left(A^T\right) = 5$.

**B.** The dimension of the column space of $A$ is 2.

**C.** $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

**D.** The rows of $A$ are linearly independent.

**E.** The columns of $A$ are linearly independent.

**11.** The vectors $(1,\ 0,\ 1,\ 2)$, $(-2,\ 0,\ a,\ 2a)$, $(1,\ 2,\ 3,\ 4)$ and $(2,\ 0,\ 2,\ a)$ are linearly independent

**A.** when $a = 4$ and $a = -2$

**B.** when $a = -2$

**C.** for all $a \neq 4$

**D.** for all values of $a$

**E.** when $a \neq 4$ and $a \neq -2$

**12.** Which of the following subsets $S$ are subspaces of the given vector space $V$?

(i) $V = \mathbb{R}^2$, and $S = \{(x, y) : xy = 0\}$

(ii) $V = \mathcal{P}_3$, and $S = \{f \in \mathcal{P}_3 : f(1) + f(-1) = 0\}$

(iii) $V = M_2 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\}$, and $S = \{A \in M_2 : \det A = 1\}$

(iv) $V = \mathbb{R}^3$, and $S = \{(x, y, z) : 5x + 2y = z\}$

(v) $V = C^2(I)$, and $S = \{f \in V : f''(t) + f(t) - 1 = 0\}$

**A.** (ii), (iv), (v)

**B.** (i), (ii), (iv)

**C.** (ii) and (iv)

**D.** (iv) and (v)

**E.** (iii) and (iv)

**13.** Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^7$ be the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$, where $A$ is a $7 \times 3$ matrix. Suppose that the Kernel of $T$ is spanned by the five vectors

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}.$$

If $p = \dim \mathsf{Ker}(T)$ and $r = \dim \mathsf{Rng}(T)$, then

**A.** $p = 3, \quad r = 0$

**B.** $p = 2, \quad r = 1$

**C.** $p = 1, \quad r = 2$

**D.** $p = 2, \quad r = 3$

**E.** $p = 3, \quad r = 2$

**14.** If $A = \begin{bmatrix} 4 & 4 & k^2 \\ 0 & 1 & 1 \\ 0 & -1 & 2k \\ 0 & 2 & 2 \end{bmatrix}$, for what value(s) of $k$ will the dimension of the nullspace of $A$ be 0 ?

**A.** $k = \frac{1}{2}$

**B.** $k \neq 2$

**C.** $k \neq -\frac{1}{2}$

**D.** $k = 1$

**E.** $k = -\frac{1}{2}$

**15.** A basis for the kernel of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$$T(x_1, x_2, x_3) = (x_1 - 2x_2 + x_3,\ x_2 - 5x_3)$$

is

**A.** $\{(1,0),\ (0,1)\}$

**B.** $\{(1,0),\ (-2,1),\ (1,5)\}$

**C.** $\{(1,-2,1),\ (0,1,5)\}$

**D.** $\{(9,5,1)\}$

**E.** $\{(1,5,9)\}$

**16.** The sum of the eigenvalues of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 2 \\ 0 & 3 & 1 \end{bmatrix}$ is

**A.** 2

**B.** 1

**C.** 0

**D.** $-1$

**E.** $-2$

**17.** Which of the following matrices are **defective** ?

$$A = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & 1 \\ 0 & -2 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

**A.** only $B$

**B.** only $A$

**C.** only $C$

**D.** both $A$ and $B$

**E.** both $A$ and $C$

**18.** Let $y(x)$ be the solution to the initial value problem $\begin{cases} y'' - 2y' + 5y = 0 \\ y(0) = 2,\ y'(0) = \alpha \end{cases}$.

Find the value of $\alpha$ such that $y(\frac{\pi}{4}) = 0$.

**A.** $\alpha = 0$

**B.** $\alpha = -1$

**C.** $\alpha = 1$

**D.** $\alpha = -2$

**E.** $\alpha = 2$

**19.** A particular solution of $(D-3)^2\,(D^2+2D+2)y = e^{3x} - e^{-x}\sin x$ has the form:

**A.** $y(x) = A_1 e^{3x} - A_2 e^{-x}\sin x$

**B.** $y(x) = A_1 e^{3x} + A_2 e^{-x}\cos x + A_3 e^{-x}\sin x$

**C.** $y(x) = A_1 x^2 e^{3x} + A_2 x e^{-x}\cos x + A_3 x e^{-x}\sin x$

**D.** $y(x) = A_1 x e^{3x} + A_2 x e^{-x}\cos x + A_3 x e^{-x}\sin x$

**E.** $y(x) = A_1 x^2 e^{3x} + A_2 e^{-x}\cos x + A_3 e^{-x}\sin x$

**20.** Find the general solution of $(D-2)(D-3)y = 7e^{2x}$.

**A.** $y(x) = c_1 e^{2x} + c_2 e^{3x} + e^{2x}$

**B.** $y(x) = c_1 e^{2x} + c_2 e^{3x} + 7e^{2x}$

**C.** $y(x) = c_1 e^{2x} + c_2 e^{3x} + x e^{2x}$

**D.** $y(x) = c_1 e^{2x} + c_2 e^{3x} - 7x e^{2x}$

**E.** $y(x) = c_1 e^{2x} + c_2 e^{3x} + c_3 e^{2x}$

**21.** Consider the spring-mass system whose motion is governed by the initial-value problem

$$y'' + 4y = 0, \ y(0) = 2, \ y'(0) = 4.$$

Find the amplitude of the motion.

**A.** $A_0 = 1$

**B.** $A_0 = 2$

**C.** $A_0 = 2\sqrt{2}$

**D.** $A_0 = \sqrt{2}$

**E.** $A_0 = \dfrac{\sqrt{2}}{2}$

**22.** Given that $y_1(x) = x$ is one solution to $x^2y'' - 4x\,y' + 4\,y = 0$, if we seek a second solution $y_2(x) = u(x)y_1(x)$ by *Reduction of Order*, then one such function $u(x)$ is

**A.** $u(x) = 1$

**B.** $u(x) = \dfrac{1}{3}\,x^3$

**C.** $u(x) = \dfrac{1}{2}\,x^2$

**D.** $u(x) = \dfrac{2}{x}$

**E.** $u(x) = 2\,\ln x$

**23.** Given that $y_1(x) = e^x$ and $y_2(x) = xe^x$ are two linearly independent solutions to the homogeneous equation $(D-1)^2\,y = 0$, using the method of *Variation of Parameters*, a particular solution to the nonhomogeneous equation

$$(D-1)^2\,y = \frac{3e^{2x}}{x^2}, \qquad (x > 0)$$

has the form $y_p = u_1(x)e^x + u_2(x)xe^x$. Which of the following are satisfied by $u_1'(x)$ and $u_2'(x)$?

**A.** $u_1' = -\dfrac{3e^x}{x}, \quad u_2' = \dfrac{3e^x}{x}$

**B.** $u_1' = \dfrac{3e^x}{x}, \quad u_2' = -\dfrac{3e^{-x}}{x^2}$

**C.** $u_1' = -\dfrac{3e^x}{x}, \quad u_2' = \dfrac{3e^x}{2x^2}$

**D.** $u_1' = -\dfrac{3e^{2x}}{x}, \quad u_2' = \dfrac{3e^x}{x^2}$

**E.** $u_1' = -\dfrac{3e^x}{x}, \quad u_2' = \dfrac{3e^x}{x^2}$

**24.** Determine the general solution of the system of differential equations $\mathbf{x}' = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix} \mathbf{x}$.

**A.** $c_1\,e^{-t}\begin{bmatrix}1\\1\end{bmatrix} + c_2\,e^{-2t}\begin{bmatrix}1\\2\end{bmatrix}$

**B.** $c_1\,e^{t}\begin{bmatrix}1\\1\end{bmatrix} + c_2\,e^{2t}\begin{bmatrix}3\\2\end{bmatrix}$

**C.** $c_1\,e^{-t}\begin{bmatrix}1\\1\end{bmatrix} + c_2\,e^{-2t}\begin{bmatrix}2\\3\end{bmatrix}$

**D.** $c_1\,e^{t}\begin{bmatrix}1\\1\end{bmatrix} + c_2\,e^{-2t}\begin{bmatrix}1\\3\end{bmatrix}$

**E.** $e^{t}\begin{bmatrix}1\\1\end{bmatrix} + e^{-t}\begin{bmatrix}1\\-1\end{bmatrix}$

**25.** Consider the nonhomogeneous linear system $\mathbf{x}' = A\mathbf{x} + \mathbf{b}(t)$, where $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$ and $\mathbf{b}(t) = \begin{bmatrix} 4\,e^{-t} \\ 0 \end{bmatrix}$. Given that the matrix $A$ has two eigenvalues $\lambda_1 = -1$ and $\lambda_2 = -3$ with the corresponding eigenvectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, respectively, then a particular solution to the nonhomogeneous system is

**A.** $\mathbf{x}_p(t) = \begin{bmatrix} e^{-t}\,(2t+1) \\ 0 \end{bmatrix}$

**B.** $\mathbf{x}_p(t) = \begin{bmatrix} e^{-t}\,(2t+1) \\ e^{-t}\,(2t-1) \end{bmatrix}$

**C.** $\mathbf{x}_p(t) = \begin{bmatrix} e^{-t}\,(2t-1) \\ 2te^{-t} \end{bmatrix}$

**D.** $\mathbf{x}_p(t) = \begin{bmatrix} e^{-t}\,(t+1) \\ e^{-t}\,(t-2) \end{bmatrix}$

**E.** $\mathbf{x}_p(t) = \begin{bmatrix} 4e^{-t} \\ t \end{bmatrix}$