MA 262, Spring 2019, Final Exam
GREEN 01

INSTRUCTIONS

1. **Switch off your phone upon entering the exam room.**
2. **Do not open the exam booklet until you are instructed to do so.**
3. **Before you open the booklet, fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
4. **MARK YOUR TEST NUMBER ON THE SCANTRON**
5. Once you are allowed to open the exam, make sure you have a complete test. There are 13 different test pages with a total of 25 problems, plus this cover page. **Each problem is worth 8 points with a total of 200 points.**
6. Do any necessary work for each problem on the space provided, or on the back of the pages of this booklet. Circle your answers in the booklet.
7. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY

1. Do not leave the exam during the first 20 minutes of the exam.
2. No talking. Do not seek or obtain any kind of help from anyone to answer the problems on the exam. If you need assistance, consult an instructor.
3. Do not look at the exam of another student. You may not compare answers with other students until your exam is finished and turned in, and then only after you have left the room.
4. Your bags must be closed throughout the exam period.
5. Notes, books, calculators and phones must be in your bags and cannot be used.
6. Do not handle phones or cameras or any other electronic device until you have finished and turned in your exam, and then only if you have left the room.
7. When time is called, all students must put down their writing instruments immediately.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME ______________________________

STUDENT SIGNATURE ______________________________

STUDENT PUID ______________________________

SECTION NUMBER ______________________________

RECITATION INSTRUCTOR ______________________________

1. Let $y(x)$ be the solution to the initial value problem

$$xy' + y = 3x^2 + 2x - 4 \qquad y(1) = 0.$$

Then $y(2) =$

A. 1

B. 2

C. 3

D. 4

E. 5

2. Let $y(x)$ be the solution to the initial value problem

$$\frac{dy}{dx} = \frac{4y - 3x}{x}, \qquad y(1) = 2.$$

Then $y(2) =$

A. 18

B. 17

C. 16

D. 15

E. 14

3. Which of the following is the solution to the initial value problem

$$(e^x \sin y - 2x - y) + (e^x \cos y - x + 3y^2)\frac{dy}{dx} = 0, \quad y(0) = \pi?$$

A. $e^x \sin y - x^2 - xy + y^3 = \pi$

B. $e^x \cos y - x^2 - xy + y^3 = -1 + \pi^3$

C. $e^x \sin y - x^2 - xy + 3y^3 = 3\pi^3$

D. $e^x \cos y + x^2 - xy + y^3 = \pi^3$

E. $e^x \sin y - x^2 - xy + y^3 = \pi^3$

4. A container initially contains 10 L of water in which 20 g of salt is dissolved. A solution containing 4 g/L of salt is pumped into the tank at a rate of 2 L/min and the well stirred mixture runs out of the tank at a rate of 1L/min. What is the concentration of the salt in the tank after 10 minutes?

A. 1.5

B. 2.5

C. 3.5

D. 4.5

E. 5.5

5. Consider the equation

$$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{y}.$$

If $v = y^2$, then v satisfies

A. $\dfrac{dv}{dx} + \dfrac{1}{x}v = e^x$

B. $\dfrac{dv}{dx} + \dfrac{2}{x}v = e^x$

C. $\dfrac{dv}{dx} + \dfrac{1}{x}v = 2e^x$

D. $\dfrac{dv}{dx} + \dfrac{2}{x}v = 2e^x$

E. $2\dfrac{dv}{dx} + \dfrac{1}{x}v = e^x$

6. Consider the system of linear equations

$$\begin{aligned} x - y + \quad z &= 2 \\ x + 4y + \quad 2z &= 2 \\ 2x + 3y + (a^2 - 1)z &= a + 2. \end{aligned}$$

Which of the following statements is true?

A. If $a = -2$ then the system is consistent.

B. If $a = 3$ then the system is inconsistent.

C. If $a = 1$ then the system has infinitely many solutions.

D. If $a = -1$ then the system has at least two distinct solutions.

E. If $a = 2$ then the system has infinitely many solutions.

7. The Wronskian of the functions $\{x, \sin x, \cos x\}$ is

A. $-x$

B. x

C. 0

D. $x(\sin^2 x - \cos^2 x)$

E. $x(\cos^2 x - \sin^2 x)$

8. Find a basis for the null space of $A = \begin{bmatrix} 1 & -2 & 2 & 3 \\ 2 & -4 & 5 & 7 \\ 3 & -6 & 3 & 6 \end{bmatrix}$.

A. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix} \right\}$.

B. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$.

C. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}$.

D. $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$.

E. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$.

9. Compute the determinant of the matrix A where

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 5 & 0 & 2 \\ 0 & 0 & 3 & 0 \end{bmatrix}.$$

A. -6

B. -4

C. 4

D. 6

E. 8

10. Let $T : \mathbb{P}_2 \to \mathbb{R}^2$ be the linear transformation satisfying $T(x^2+x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $T(2x+1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $T(2) = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$. Find $T(x^2)$.

A. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

E. $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

11. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 4 & 1 \end{bmatrix}$. Given $\det A=3$, find the (1,2) entry of A^{-1}.

A. -2

B. -1

C. 0

D. 1

E. 2

12. Which of the following are vector spaces?

(i) $\{(x, y) \in \mathbb{R}^2 : \ x > y\}$

(ii) $\{(x, y) \in \mathbb{R}^2 : \ 2x - 3y = 0\}$

(iii) All solutions to $y'' + 2y = 0$ on $(-\infty, \infty)$.

(iv) All solutions to $y'' + 2 = 0$ on $(-\infty, \infty)$.

A. (i), (ii), and (iii)

B. (ii), (iii), and (iv)

C. (ii) and (iii)

D. (i) and (iv)

E. None of them are vector spaces.

13. Which of the following is a basis for Span$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} \right\}$?

A. $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

14. Given that A and B are 4×4 matrices, $\det A=2$, and $\det(2A^{-2}B^T)=1$, find $\det B$.

A. $\frac{1}{8}$

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. 2

E. 4

15. For the equation:

$$y'' + y' - 2y = 2e^t,$$

the particular solution y_p is given by

A. $y_p = \frac{4}{3}te^t$

B. $y_p = 2e^t$

C. $y_p = -te^t$

D. $y_p = -\frac{1}{2}te^t$

E. $y_p = \frac{2}{3}te^t$

16. Let A be the matrix defined by

$$A = \begin{bmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

Count the number of true statements about A among those:

(i) The vector $\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ is an eigenvector for the eigenvalue $\lambda = 2$.

(ii) $1 + i$ and $1 - i$ are eigenvalues of A.

(iii) -1 and 4 are eigenvalues of A.

(iv) A has 3 linearly independent eigenvectors.

A. 0

B. 1

C. 2

D. 3

E. 4

17. For the matrix A given by

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & -2 \\ 1 & 8 & -3 \end{bmatrix},$$

we have $2\dim(\mathrm{Null}(A)) - \mathrm{rank}(A) =$

A. -3

B. 0

C. 2

D. 3

E. 6

18. The general solution to the homogeneous equation $t^2y'' + 7ty' + 5y = 0$ on the interval $0 < t < \infty$ is $y(t) = c_1t^{-1} + c_2t^{-5}$. A particular solution to the nonhomogeneous equation $t^2y'' + 7ty' + 5y = t$ has the form $y_p(t) = u_1(t)t^{-1} + u_2(t)t^{-5}$. Which of the following are satisfied by u_1' and u_2'?

A. $u_1' = \frac{t}{4}, \quad u_2' = -\frac{t^5}{4}$

B. $u_1' = \frac{t^3}{4}, \quad u_2' = -\frac{t^7}{4}$

C. $u_1' = -t^7, \quad u_2' = t^3$

D. $u_1' = t^5, \quad u_2' = t$

E. $u_1' = -t^5, \quad u_2' = t$

19. A mass-spring system is given by the initial value problem: $y'' + 3y' + 2y = 0$, $y(0) = 2$, $y'(0) = -5$, where t is the number of seconds after the motion begins. How many seconds does it take for the mass to cross its equilibrium position for the first time?

A. $\ln 2$

B. $\ln 3$

C. $\ln 6$

D. 2

E. 3

20. Given that the function $y_1(t) = t$ is a solution to the differential equation

$$t^2 y'' + 2ty' - 2y = 0, \quad t > 0$$

choose a function y_2 from the list below so that the pair $\{y_1, y_2\}$ form a fundamental set of solutions to the differential equation above.

A. $t - \ln(t + 4)$

B. $t - \sin(t^2)$

C. $2t + \frac{1}{t^2}$

D. $t - e^t$

E. $\frac{4}{t^4}$

21. A general solution of $D^2(D^2 - 8D + 25)^2 y = 0$ is:

A. $C_1 x + C_2 x^2 + C_3 e^{4x} \cos 3x + C_4 e^{4x} \sin 3x$

B. $C_1 + C_2 x + C_3 e^{4x} \cos 3x + C_4 e^{4x} \sin 3x$

C. $C_1 + C_2 x + C_3 e^{-4x} \cos 3x + C_4 e^{-4x} \sin 3x + C_5 x e^{-4x} \cos 3x + C_6 x e^{-4x} \sin 3x$

D. $C_1 + C_2 x + C_3 e^{4x} \cos 3x + C_4 e^{4x} \sin 3x + C_5 x e^{4x} \cos 3x + C_6 x e^{4x} \sin 3x$

E. $C_1 x + C_2 x^2 + C_3 e^{4x} \cos 3x + C_4 e^{4x} \sin 3x + C_5 x e^{4x} \cos 3x + C_6 x e^{4x} \sin 3x$

22. If we use the method of undetermined coefficients, the form of a particular solution to

$$y''' - y'' - y' + y = e^x + 2x^2$$

is:

A. $Ae^x + Bx^2 + Cx + D$

B. $Axe^x + Bx^2 + Cx + D$

C. $Ax^2e^x + Bx^2 + Cx + D$

D. $Ax^2e^x + Bx^2$

E. $Axe^x + Bx^2$

23. Which one of the following is an annihilator of the function $f(x) = e^x x^2 + \sin 3x$?

A. $(D-1)D^2 + (D^2+9)$

B. $(D-1)D^2(D^2+9)$

C. $(D-1)^2(D^2+9)$

D. $(D-1)^3(D^2+9)$

E. $(D-1)^3 + (D^2+9)$

24. Let $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ be the solution of the initial value problem

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}.$$

Find $\begin{bmatrix} x_1(1) \\ x_2(1) \end{bmatrix}$.

A. $\begin{bmatrix} 7e^2 - 3e^3 \\ -7e^2 + 6e^3 \end{bmatrix}$

B. $\begin{bmatrix} -7e^2 + 3e^3 \\ 7e^2 - 6e^3 \end{bmatrix}$

C. $\begin{bmatrix} e^2 + e^3 \\ -e^2 - 2e^3 \end{bmatrix}$

D. $\begin{bmatrix} -e^2 - e^3 \\ e^2 + 2e^3 \end{bmatrix}$

E. $\begin{bmatrix} -2e^{-4} - 3e \\ 2e^{-4} + 6e \end{bmatrix}$

25. If a fundamental matrix for $\mathbf{x}' = \mathbf{A}\mathbf{x}(t)$ is $X(t) = \begin{bmatrix} -e^{-t} & e^{5t} \\ e^{-t} & 2e^{5t} \end{bmatrix}$, then the general solution to the system $\mathbf{x}' = \mathbf{A}\mathbf{x}(t) + \begin{bmatrix} 12e^{t} \\ 0 \end{bmatrix}$ is:

A. $\begin{bmatrix} -e^{-t} & e^{5t} \\ e^{-t} & 2e^{5t} \end{bmatrix} \left(\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} 4e^{2t} \\ -e^{-4t} \end{bmatrix} \right)$

B. $\begin{bmatrix} -e^{-t} & e^{5t} \\ e^{-t} & 2e^{5t} \end{bmatrix} \left(\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} -4e^{2t} \\ -e^{-4t} \end{bmatrix} \right)$

C. $\begin{bmatrix} -e^{-t} & e^{5t} \\ e^{-t} & 2e^{5t} \end{bmatrix} \left(\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} 4e^{-4t} \\ -4e^{2t} \end{bmatrix} \right)$

D. $\begin{bmatrix} -e^{-t} & e^{5t} \\ e^{-t} & 2e^{5t} \end{bmatrix} \left(\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} e^{2t} \\ 4e^{-4t} \end{bmatrix} \right)$

E. $\begin{bmatrix} -e^{-t} & e^{5t} \\ e^{-t} & 2e^{5t} \end{bmatrix} \left(\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} 4e^{2t} \\ e^{-4t} \end{bmatrix} \right)$