MA 265, Fall 2022, Midterm I (GREEN)

INSTRUCTIONS:

1. Write your answers of the seven multiple choice questions into the table on the last page. Show all your work on the questions and you may use the back of the test pages as scratch paper if needed.

2. After you have finished the exam, hand in your test booklet to your instructor.

<table>
<thead>
<tr>
<th>101</th>
<th>MWF</th>
<th>10:30AM</th>
<th>Ying Zhang</th>
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<tr>
<td>102</td>
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<td>Daniel Tsun-Dan Le</td>
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<td>Yiran Wang</td>
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<td>410</td>
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<td>Arun Debray</td>
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<td>501</td>
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<td>Vaibhav Pandey</td>
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<td>502</td>
<td>MWF</td>
<td>10:30AM</td>
<td>Ayan Maiti</td>
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| 600  | MWF   | 1:30PM  | Seongjun Choi |
| 601  | MWF   | 1:30PM  | Ayan Maiti   |
| 650  | MWF   | 10:30AM | Yevgeniya Tarasova |
| 651  | MWF   | 9:30AM  | Yevgeniya Tarasova |
| 701  | MWF   | 3:30PM  | Seongjun Choi |
| 702  | MWF   | 11:30AM | Yiran Wang   |
| 703  | MWF   | 12:30PM | Ke Wu        |
| 704  | MWF   | 1:30PM  | Ke Wu        |
| 705  | MWF   | 12:30PM | Seongjun Choi |
| 706  | TR    | 1:30PM  | Vaibhav Pandey |
| 707  | MWF   | 3:30PM  | Siamak Yassemi |
| 708  | MWF   | 2:30PM  | Siamak Yassemi |

3. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices.

4. When time is called, all students must put down their writing instruments immediately. You may remain in your seat while your instructor will collect the exam booklets.

5. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above instructions regarding academic dishonesty:

STUDENT NAME

STUDENT SIGNATURE

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SECTION NUMBER
1. (10 points) Let \( A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \), \( B = \begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix} \), and \( C = AB^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \), then \( a + b + c + d = \)

A. \(-7\)
B. \(8\)
C. \(7\)
D. \(-8\)
E. \(0\)

2. (10 points) Let \( L \) be a linear transformation from \( \mathbb{R}^3 \) to \( \mathbb{R}^3 \) whose standard matrix is
\[
\begin{bmatrix}
1 & 2 & 3 \\
0 & 1 & 1 \\
2 & 3 & k
\end{bmatrix}
\]
where \( k \) is a real number. Find all values of \( k \) such that \( L \) is one-to-one.

A. \( k \neq 1 \)
B. \( k \neq 2 \)
C. \( k \neq 3 \)
D. \( k \neq 4 \)
E. \( k \neq 5 \)
3. (10 points) Which of the following statements is/are always TRUE?

(i) If \( A \) is a singular \( 8 \times 8 \) matrix, then its last column must be a linear combination of the first seven columns.

(ii) Let \( A \) be a \( 5 \times 7 \) matrix such that \( A \cdot x = b \) is consistent for any \( b \in \mathbb{R}^5 \), and let \( B \) be a \( 7 \times 11 \) matrix such that \( B \cdot x = c \) is consistent for any \( c \in \mathbb{R}^7 \). Then, the matrix equation \( AB \cdot x = b \) is consistent for any \( b \in \mathbb{R}^5 \).

(iii) For any \( m \times n \) matrix \( A \), the dimension of the null space of \( A \) equals the dimension of the null space of its transpose \( A^T \).

(iv) If \( A \) is an \( m \times n \) matrix, then the set \( \{ A \cdot x | x \in \mathbb{R}^n \} \) is a subspace of \( \mathbb{R}^m \).

A. (i) only
B. (i) and (ii) only
C. (iv) only
D. (ii) and (iv) only
E. (iii) and (iv) only

4. (10 points) Compute the determinant of the given matrix

\[
\begin{bmatrix}
5 & -7 & 2 & 2 \\
0 & 3 & 0 & -4 \\
-5 & -8 & 0 & 3 \\
0 & 5 & 0 & -6
\end{bmatrix}
\]

A. \(-20\)
B. \(20\)
C. \(18\)
D. \(2\)
E. \(0\)
5. (10 points) Which of the following statements is always TRUE?

A. If $A$ is an $n \times n$ matrix with all entries being positive, then $\det(A) > 0$.
B. If $A$ and $B$ are two $n \times n$ matrices with $\det(A) > 0$ and $\det(B) > 0$, then also $\det(A + B) > 0$.
C. If $A$ and $B$ are two $n \times n$ matrices such that $AB = 0$, then both $A$ and $B$ are singular.
D. If rows of an $n \times n$ matrix $A$ are linearly independent, then $\det(A^TA) > 0$.
E. If $A$ is an $n \times n$ matrix with $A^2 = I_n$, then $\det(A) = 1$.

6. (10 points) Let $A = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 6 & 3 \\ 3 & 8 & 10 \end{bmatrix}$ and let its inverse $A^{-1} = [b_{ij}]$. Find $b_{12}$.

A. 14
B. -14
C. 1
D. -1
E. 6
7. (10 points) Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$, $v_2 = \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix}$ and $x = \begin{bmatrix} -4 \\ -5 \\ 13 \end{bmatrix}$, and $B = \{v_1, v_2\}$. Then $B$ is a basis for $H = \text{span}\{v_1, v_2\}$. Determine if $x$ is in $H$, and if it is, find the coordinate vector of $x$ relative to $B$.

A. $[x]_B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

B. $[x]_B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

C. $[x]_B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

D. $[x]_B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

E. $[x]_B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$
8. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the linear transformation for which

$$T\left( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \quad \text{and} \quad T\left( \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}. $$

(4 points)(1) Let $A$ be the standard matrix of $T$, find $A$.

(2 points)(2) Find the image of the vector $\mathbf{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

(4 points)(3) Is the vector $\mathbf{b} = \begin{bmatrix} 0 \\ -2 \\ 5 \end{bmatrix}$ in the range of $T$? If so, find all the vectors $\mathbf{x}$ in $\mathbb{R}^2$ such that $T(\mathbf{x}) = \mathbf{b}$. 
9. Consider the linear system

\[
\begin{align*}
x + 2y + 3z &= 2 \\
y + az &= -4 \\
2x + 5y + a^2z &= a - 3
\end{align*}
\]

(4 points)(1) Find a row echelon form for the augmented matrix of the system.

(2 points)(2) For which value(s) of \( a \) does this system have an infinite number of solutions?

(2 points)(3) For which value(s) of \( a \) does this system have no solution?

(2 points)(4) For which value(s) of \( a \) does this system have a unique solution?
10. Let

\[ A = \begin{bmatrix}
1 & 2 & 0 & -1 & 2 \\
2 & 3 & 1 & -3 & 7 \\
3 & 4 & 1 & -3 & 9
\end{bmatrix}. \]

(5 points) (1) Find the REDUCED row echelon form for the matrix \( A \).

(5 points) (2) Find a basis for the null space of \( A \).
Please write your answers of the 7 multiple choice questions in the following table.

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
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Total Points: ___________________________