MA 265, Fall 2023, Midterm I (GREEN)

INSTRUCTIONS:

1. **Write your answers of the seven multiple choice questions into the table on the last page. Show all your work on the questions and you may use the back of the test pages as scratch paper if needed.**
2. After you have finished the exam, hand in your test booklet to your instructor.

101	MWF	12:30PM	Ying Zhang	600	MWF	10:30AM	Farrah Yhee
102	MWF	11:30AM	Ying Zhang	601	MWF	11:30AM	Farrah Yhee
153	MWF	1:30PM	Ying Zhang	650	MWF	11:30AM	Yiran Wang
154	MWF	11:30AM	Takumi Murayama	651	MWF	12:30PM	Yiran Wang
205	MWF	11:30AM	Jing Wang	701	TR	12:00PM	Raechel Polak
206	MWF	2:30PM	Sam Nariman	702	TR	4:30PM	Vaibhav Pandey
357	MWF	8:30AM	Jonathon Peterson	703	MWF	10:30AM	Takumi Murayama
410	MWF	11:30AM	Siamak Yassemi	704	MWF	8:30PM	Anurag Sahay
451	MWF	1:30PM	Siamak Yassemi	705	MWF	1:30PM	Yi Wang
501	MWF	1:30PM	Yilong Zhang	706	MWF	12:30PM	Yi Wang
502	MWF	10:30AM	Yilong Zhang	707	TR	3:00PM	Vaibhav Pandey

3. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices.
4. When time is called, all students must put down their writing instruments immediately. You may remain in your seat while your instructor will collect the exam booklets.
5. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above instructions regarding academic dishonesty:

STUDENT NAME ______________________

STUDENT SIGNATURE ______________________

STUDENT PUID ______________________

SECTION NUMBER ______________________

1. (10 points) Let $A = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 5 \end{bmatrix}$, and $C = B^{-1}A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $a+b+c+d =$

A. -3

B. 3

C. 1

D. -1

E. 0

2. (10 points) Let A be an $m \times n$ matrix. Let R be the *reduced* row echelon form of A. Which of the following statements must be **TRUE**?

(i) $\mathrm{Nul}(A) = \mathrm{Nul}(R)$.

(ii) $\mathrm{Col}(A) = \mathrm{Col}(R)$.

(iii) $\mathrm{Rank}(A) = \mathrm{Rank}(R)$.

(iv) If A has a pivot position in every column, then $A\mathbf{x} = \mathbf{b}$ is consistent for any $\mathbf{b}$ in $\mathbb{R}^m$.

(v) If A is a square matrix and $\det(R) = 0$, then $\det(A) = 0$.

A. (i), (iii), (v)

B. (i), (ii), (iv), (v)

C. (i), (iii)

D. (ii), (iii), (v)

E. (i), (iii), (iv), (v)

3. (10 points) Let L be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ whose standard matrix is

$$\begin{bmatrix} 1 & a & a+1 \\ 2 & a+2 & a-1 \\ 2-a & 0 & 0 \end{bmatrix}$$

where a is a real number. Find all values of a such that L is not one-to-one.

A. 2 only

B. 2 or $\frac{1}{2}$

C. 2 or $-\frac{1}{2}$

D. 2 or -2

E. 2 or 0

4. (10 points) Let $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 4 \\ 1 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 3 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 3 \\ 7 \\ 2 \\ 6 \end{bmatrix}$, which of the following statements is always true?

A. $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ are linearly dependent.

B. $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ span $\mathbb{R}^4$.

C. Any vector in $\mathbb{R}^4$ can be written as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

D. Let A be the matrix whose column vectors are given by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$. Then $A\mathbf{x} = \mathbf{b}$ has a unique solution for any vector $\mathbf{b}$ in $\mathbb{R}^4$.

E. Let A be the matrix whose column vectors are given by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$. Then $A\mathbf{x} = \mathbf{0}$ always has a unique solution.

5. (10 points) Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a linear transformation with standard matrix A. Which of the following statements must be **TRUE**?

A. A is an $n \times m$ matrix.

B. If T is one-to-one, then $A\mathbf{x} = \mathbf{b}$ has a unique solution for any $\mathbf{b}$ in $\mathbb{R}^m$.

C. If T is one-to-one, then $m \geq n$.

D. The columns of A are linearly independent if and only if $\text{rank}(A) = m$.

E. If the set $\{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_k}\}$ spans $\mathbb{R}^n$, then the set $\{T(\mathbf{v_1}), T(\mathbf{v_2}), \ldots, T(\mathbf{v_k})\}$ spans $\mathbb{R}^m$.

6. (10 points) Let $\boldsymbol{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$, $\boldsymbol{v}_2 = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$ and $\boldsymbol{x} = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$, and $\boldsymbol{B} = \{\boldsymbol{v}_1, \boldsymbol{v}_2\}$. Then $\boldsymbol{B}$ is a basis for $H = \text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2\}$. Determine if $\boldsymbol{x}$ is in H, and if it is, find the coordinate vector of $\boldsymbol{x}$ relative to $\boldsymbol{B}$.

A. Yes, $\boldsymbol{x}$ is in H, and $[\boldsymbol{x}]_{\boldsymbol{B}} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$

B. Yes, $\boldsymbol{x}$ is in H, and $[\boldsymbol{x}]_{\boldsymbol{B}} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

C. Yes, $\boldsymbol{x}$ is in H, and $[\boldsymbol{x}]_{\boldsymbol{B}} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

D. Yes, $\boldsymbol{x}$ is in H, and $[\boldsymbol{x}]_{\boldsymbol{B}} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

E. No, $\boldsymbol{x}$ is not in H.

7. (10 points) Which of the following collection of vectors is linearly independent?

A. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 0 \\ 9 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 2 \\ 4 \\ -3 \\ -5 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ -6 \\ -10 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \\ 0 \\ 7 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 3 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

8. Consider the linear system

$$\begin{bmatrix} 1 & 1 & a \\ 0 & 1 & a^2-2 \\ 3 & 2 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ a \\ 3 \end{bmatrix}.$$

(4 points)(1) Find a row echelon form for the augmented matrix of the system.

(2 points)(2) For which value(s) of a does this system have an infinite number of solutions?

(2 points)(3) For which value(s) of a does this system have no solution?

(2 points)(4) For which value(s) of a does this system have a unique solution?

9. Let

$$A = \begin{bmatrix} 1 & 0 & -1 & -2 & 3 \\ 2 & 0 & -3 & -4 & 5 \\ 5 & 0 & -6 & -10 & 14 \end{bmatrix}.$$

(5 points)(1) Find the REDUCED row echelon form for the matrix A.

(5 points)(2) Find a basis for the null space of A.

10. Consider the given matrix $A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 \end{bmatrix}$.

(5 points)(1) Find the determinant of matrix A by expanding along the first column.

(5 points)(2) Let matrix B be the inverse matrix of the matrix A, and $B = [b_{ij}]$. Find b_{32}.

Please write your answers of the 7 multiple choice questions in the following table.

Question	Answer
1. (10 points)	
2. (10 points)	
3. (10 points)	
4. (10 points)	
5. (10 points)	
6. (10 points)	
7. (10 points)	

Total Points: ______________________