MA 265, Fall 2024, Midterm I (Green)

INSTRUCTIONS:

- 1. Write your answers of the seven multiple choice questions into the table on the last page. Show all your work on the questions and you may use the back of the test pages as scratch paper if needed.
- 2. After you have finished the exam, hand in your test booklet to your instructor.

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- 3. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices.
- 4. When time is called, all students must put down their writing instruments immediately. You may remain in your seat while your instructor will collect the exam booklets.
- 5. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above instructions regarding academic dishonesty:

STUDENT NAME	
STUDENT SIGNATURE	
STUDENT PUID	
SECTION NUMBER	

1. (10 points) Let $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$. Find the matrix *B* that satisfies the following equation:

$$B^T A^{-1} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}.$$

A. $\begin{bmatrix} 0 & 2 \\ 2 & -2 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 2 \\ 1 & -2 \end{bmatrix}$ C. $\begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$ D. $\begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$ E. $\begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$

		a	b	c		a + 5g	c+5i	3b + 15h	
2.	(10 points) Suppose that det	d	e	f	= 3. What is det	d	f	3e	?
		$\lfloor g$	h	i		$\int g$	i	3h	

- A. -3
- B. -15
- C. 15
- D. -9
- E. 9

- **3.** (10 points) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation with the standard matrix A. Which of the following statements is always TRUE?
 - A. If T is onto, then the columns of A are linearly independent.
 - B. If T is one-to-one, then the columns of A span \mathbb{R}^m .
 - C. If $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_k)\}$ is a basis of \mathbb{R}^m , then the set $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is a basis of \mathbb{R}^n .
 - D. If $T(\mathbf{x}) = \mathbf{b}$ has more than one solution for some $\mathbf{b} \in \mathbb{R}^m$, then rank of A is less than m.
 - E. If rank of A equals n and T is onto, then T is also one-to-one.

4. (10 points) Consider the following three vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1\\0\\x \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1\\1\\x \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} x\\0\\1 \end{bmatrix}$$

For what values of x are these vectors linearly independent?

- A. $x \neq 1$ only
- B. $x \neq 2$ only
- C. $x \neq -1$ only
- D. $x \notin \{1, -1\}$
- E. $x \notin \{2, -2\}$

- 5. (10 points) Let A be a matrix. Which of the following statements must be TRUE?
 - (i) If Nul $A = \{0\}$, then the matrix transformation defined by $T(\mathbf{x}) = A\mathbf{x}$ is onto.
 - (ii) If $A\mathbf{x} = 2A\mathbf{y}$, then $\mathbf{x} = 2\mathbf{y}$.
 - (iii) If **x** is a vector \mathbb{R}^3 and $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then rank(A) = 3.
 - (iv) If A is a 3×3 matrix and $A\mathbf{x} = \begin{bmatrix} 0\\0\\0 \end{bmatrix}$ has infinitely many solutions, then the dimension of Nul A is 3.
 - (v) If A is a 3×3 matrix and $A\mathbf{x} = \mathbf{b}$ has a unique solution, then $\det(A) = 0$.
 - A. (iii) only
 - B. (i) and (iii) only
 - C. (i), (iii) and (iv) only
 - D. (ii), (iv) and (v) only
 - E. all are true
- 6. (10 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\2\end{bmatrix} \text{ and } T\left(\begin{bmatrix}a\\b\end{bmatrix}\right) = \begin{bmatrix}3\\2\end{bmatrix},$$

find a-b.

- A. 1
- B. 2
- C. 3
- D. -1
- E. 0

7. (10 points) Which of the following set of vectors spans \mathbb{R}^4 ?

A.
$$\left\{ \begin{bmatrix} 1\\2\\0\\1 \end{bmatrix}, \begin{bmatrix} 2\\2\\0\\1 \end{bmatrix}, \begin{bmatrix} -1\\-2\\0\\1 \end{bmatrix}, \begin{bmatrix} 0\\0\\0\\0 \end{bmatrix} \right\}$$

B. $\left\{ \begin{bmatrix} 3\\0\\0\\0 \end{bmatrix}, \begin{bmatrix} 2\\4\\0\\0\\0 \end{bmatrix}, \begin{bmatrix} -5\\-7\\8\\0 \end{bmatrix}, \begin{bmatrix} 5\\6\\6\\3 \end{bmatrix} \right\}$
C. $\left\{ \begin{bmatrix} 3\\2\\2\\1\\1 \end{bmatrix}, \begin{bmatrix} 2\\2\\7\\-1 \end{bmatrix}, \begin{bmatrix} 1\\-2\\5\\1 \end{bmatrix} \right\}$
D. $\left\{ \begin{bmatrix} 1\\2\\0\\1\\1 \end{bmatrix}, \begin{bmatrix} 2\\2\\0\\1\\1 \end{bmatrix}, \begin{bmatrix} -1\\-2\\0\\1\\1 \end{bmatrix}, \begin{bmatrix} 5\\6\\0\\3 \end{bmatrix} \right\}$
E. $\left\{ \begin{bmatrix} 1\\2\\1\\1\\1 \end{bmatrix}, \begin{bmatrix} 2\\2\\0\\1\\1 \end{bmatrix}, \begin{bmatrix} -1\\-2\\0\\1\\1 \end{bmatrix}, \begin{bmatrix} 5\\6\\0\\3 \end{bmatrix} \right\}$

8. Let
$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 5 \\ 3 & 2 & 2 \end{bmatrix}$$
.
(5 points)(1) For what value of r is the system $A\mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ r \end{bmatrix}$ consistent?

(5 points)(2) Let $\mathcal{B} = \left\{ \begin{bmatrix} 1\\0\\3 \end{bmatrix}, \begin{bmatrix} 2\\2\\2 \end{bmatrix} \right\}$ be a basis for the column space of A. Determine if $\mathbf{b} = \begin{bmatrix} 1\\4\\-5 \end{bmatrix}$ is in the column space of A, and if it is, find the coordinate vector of \mathbf{b} relative to the ordered basis \mathcal{B} . **9.** Let

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 2 & 2 & 4 \\ 1 & 2 & 3 & 6 \\ 1 & 2 & 4 & 8 \end{bmatrix}.$$

(5 points)(1) Find the REDUCED row echelon form for the matrix A.

(5 points)(2) Find a basis for the null space of A.

10. Consider the given matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 3 & 4 & 5 \end{bmatrix}$.

(5 points)(1) Find the determinant of matrix A by expanding along the third row.

(5 points)(2) Let matrix B be the inverse matrix of matrix A, and $B = [b_{ij}]$. Find b_{14} .

Please write your answers of the 7 multiple choice questions in the following table.

Question	Answer
1. (10 points)	
2. (10 points)	
3. (10 points)	
4. (10 points)	
5. (10 points)	
6. (10 points)	
7. (10 points)	

Total Points: _____