MA 265, Spring 2022, Midterm I (GREEN)

INSTRUCTIONS:

1. **Write your answers of the seven multiple choice questions into the table on the last page. Show all your work on the questions and you may use the back of the test pages as scratch paper if needed.**
2. After you have finished the exam, hand in your test booklet to your instructor.

172	MWF	11:30AM	Zhang, Ying
173	MWF	12:30PM	Zhang, Ying
185	TR	12:00PM	Tsymbaliuk, Oleksandr
196	TR	1:30PM	Tsymbaliuk, Oleksandr
201	MWF	11:30AM	Debray, Arun
202	TR	12:00PM	Zhang, Zecheng
213	TR	4:30PM	Zhang, Zecheng
214	MWF	4:30PM	Xu, Xuefeng
225	MWF	3:30PM	Xu, Xuefeng
226	MWF	10:30AM	Yhee, Farrah
237	MWF	11:30AM	Yhee, Farrah
238	TR	9:00AM	Yang, Guang
240	TR	10:30AM	Yang, Guang
241	TR	3:00PM	Noack, Christian james
252	TR	4:30PM	Noack, Christian james
253	TR	4:30PM	Kadattur, Shuddhodan
264	TR	3:00PM	Kadattur, Shuddhodan
265	MWF	3:30PM	Nguyen, Thi-Phong
276	MWF	4:30PM	Nguyen, Thi-Phong
277	TR	12:00PM	Zhang, Qing
281	TR	1:30PM	Zhang, Qing
282	MWF	1:30PM	Tang, Shiang
283	MWF	1:30PM	Zhang, Ying
284	MWF	2:30PM	Debray, Arun
285	MWF	2:30PM	Tang, Shiang
287	TR	9:00AM	Rivera, Manuel
288	MWF	10:30AM	Mohammad-Nezhad, Ali
289	MWF	11:30AM	Mohammad-Nezhad, Ali
290	TR	10:30AM	Miller, Jeremy
291	TR	12:00PM	Ulrich, Bernd
292	MWF	3:30PM	Heinzer, William

3. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices.
4. When time is called, all students must put down their writing instruments immediately. You may remain in your seat while your instructor will collect the exam booklets.
5. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above instructions regarding academic dishonesty:

STUDENT NAME ______________________________

STUDENT SIGNATURE ______________________________

STUDENT PUID ______________________________

SECTION NUMBER ______________________________

1. (10 points) For which value of a is the following system of equations in the variables x, y, and z inconsistent?

$$\begin{aligned} x + 2y - z &= 1 \\ ax + ay + 3z &= 0 \\ y - 2z &= 2 \end{aligned}$$

A. $a = 0$

B. $a = -3$

C. $a = 3$

D. $a = 2$

E. $a = -2$

2. (10 points) Let A be an $m \times n$ matrix. Which of the following statements must be true?

(i) If equation $A\mathbf{x} = \mathbf{b}$ is consistent for each $\mathbf{b} \in \mathbb{R}^m$ then $\text{rank}(A) = n$.

(ii) If equation $A\mathbf{x} = 0$ has only trivial solution, then $\text{rank}(A) = n$.

(iii) If $\text{rank}(A) = n$ then rows of A form a linearly dependent set.

(iv) If $\text{rank}(A) = n$ and A is square matrix then A is invertible.

(v) If $\text{rank}(A) = m$ and the linear transform $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one then A is invertible.

A. (i), (ii) and (iv) only

B. (ii), (iii) and (v) only

C. (i), (ii), (iv) and (v) only

D. (ii), (iv) and (v) only

E. all are true

3. (10 points) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a one-to-one linear transformation. Let A be the matrix associated to T. Which of the following statements is not always true?

A. If $\mathbf{v_1}$ and $\mathbf{v_2}$ are linearly independent, so are $T(\mathbf{v_1})$ and $T(\mathbf{v_2})$.

B. If $T(\mathbf{v}) = \mathbf{0}$, then $\mathbf{v} = \mathbf{0}$.

C. $m \geq n$.

D. $T(\mathbf{v}+\mathbf{w})$=$T(\mathbf{v}) + T(\mathbf{w})$ for all $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$.

E. For all $\mathbf{b} \in \mathbb{R}^m$, the equation $A\mathbf{x} = \mathbf{b}$ is consistent.

4. (10 points) Which of the following statements must be TRUE?

(i) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation associated to a 2×2 matrix A and $A^T A$ is the identity matrix, then for any parallelogram S, the area of $T(S)$ is the area of S.

(ii) A homogeneous linear system of n equations in n variables has infinitely many solutions if and only if the determinant of the coefficient matrix is zero.

(iii) If A is an $n \times n$ matrix with non-zero determinant, then A is invertible and the inverse of A is given by

$$A^{-1} = \frac{1}{\det(A)} A.$$

(iv) If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is the linear transformation associated to a 3×3 matrix A such that there is some parallelepiped S for which $T(S)$ is a flat parallelogram, then $\det(A) = 0$.

A. (i) and (iv) only

B. (iii) and (iv) only

C. (i), (ii), and (iv) only

D. (ii), (iii) and (iv) only

E. all are true

5. (10 points) Which of the following collection of vectors is linearly **independent**?

A. $\left\{ \begin{bmatrix} 4 \\ 5 \\ -8 \end{bmatrix}, \begin{bmatrix} -5 \\ -6 \\ 9 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ 3 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 22 \\ 8 \\ 0 \\ 0 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -3 \\ -3 \end{bmatrix} \right\}$

6. (10 points) Assume that the determinant of the matrix $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is -2. What is the determinant of the matrix $B = \begin{bmatrix} 3a-5b+c & 3d-5e+f & 3g-5h+i \\ 2c & 2f & 2i \\ b & e & h \end{bmatrix}$?

A. -12

B. 12

C. -24

D. 6

E. -6

7. (10 points) Let $\mathcal{B} = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$, where $\mathbf{v_1} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 0 \\ 3 \\ 4 \\ 1 \end{bmatrix}$, $\mathbf{v_3} = \begin{bmatrix} -1 \\ 1 \\ -3 \\ 2 \end{bmatrix}$ is a linearly independent set. Denote $H = \text{Span}\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ which is a subspace of $\mathbb{R}^4$. Is $\mathbf{x} = \begin{bmatrix} 9 \\ -8 \\ -1 \\ -5 \end{bmatrix}$ in H? If so, find the $\mathcal{B}-$ coordinate vector of $\mathbf{x}$.

A. $\mathbf{x}$ is in H and the $\mathcal{B}-$ coordinate vector of $\mathbf{x}$ is $\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$

B. $\mathbf{x}$ is in H and the $\mathcal{B}-$ coordinate vector of $\mathbf{x}$ is $\begin{bmatrix} 2 \\ -3 \\ -3 \end{bmatrix}$

C. $\mathbf{x}$ is in H and the $\mathcal{B}-$ coordinate vector of $\mathbf{x}$ is $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

D. $\mathbf{x}$ is in H and the $\mathcal{B}-$ coordinate vector of $\mathbf{x}$ is $\begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix}$

E. $\mathbf{x}$ is not in H

8. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation with

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}1\\0\\5\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}2\\1\\6\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}3\\4\\0\end{bmatrix}.$$

(2 points)(1) Let A be the standard matrix of T, find A.

(3 points)(2) Find the image of the vector $\mathbf{u} = \begin{bmatrix}1\\2\\-1\end{bmatrix}$.

(5 points)(3) Is the vector $\mathbf{b} = \begin{bmatrix}10\\6\\27\end{bmatrix}$ in the range of T? If so, find a vector $\mathbf{x}$ in $\mathbb{R}^3$ such that $T(\mathbf{x}) = \mathbf{b}$.

9. Consider the following matrix

$$A = \begin{bmatrix} 1 & 0 & -5 & 1 & 4 \\ -2 & 1 & 6 & -2 & -2 \\ 0 & 2 & -8 & 1 & 9 \end{bmatrix}.$$

(4 points)(1) Find the REDUCED row echelon form for the matrix A.

(4 points)(2) Find a basis for the null space of A.

(2 points)(3) Find a basis for the column space of A.

10. Consider the following matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}.$$

(3 points)(1) Find the determinant of A.

(3 points)(2) Find the determinant of $2A$ and the determinant of $(2A)^{-1}$.

(4 points)(3) Let B be the inverse matrix of A, find the (2,3)-entry of matrix B.

Please write your answers of the 7 multiple choice questions in the following table.

Question	Answer
1. (10 points)	
2. (10 points)	
3. (10 points)	
4. (10 points)	
5. (10 points)	
6. (10 points)	
7. (10 points)	

Total Points: ________________________