MA 265, Fall 2024, Midterm II (Green)

INSTRUCTIONS:

1. **Write your answers of the first seven multiple choice questions into the table on the last page. Show all your work on the questions and you may use the back of the test pages as scratch paper if needed.**
2. After you have finished the exam, hand in your test booklet to your instructor.

101	MWF	11:30AM	Oleksandr Tsymbaliuk	600	MWF	8:30AM	Youssef Hakiki
102	MWF	12:30PM	Oleksandr Tsymbaliuk	650	MWF	8:30AM	Yuxi Han
153	TR	1:30PM	Yilong Zhang	701	MWF	12:30PM	Mathew George
154	TR	10:30AM	Yilong Zhang	702	MWF	12:30PM	Nour Khoudari
205	MWF	3:30PM	Soheil Memariansorkhabi	704	MWF	4:30PM	Michael Monaco
206	MWF	11:30AM	Yuxi Han	705	MWF	2:30PM	Kuan-Ting Yeh
357	MWF	11:30AM	Vaibhav Pandey	706	MWF	3:30PM	Michael Monaco
410	MWF	2:30PM	Vaibhav Pandey	707	MWF	1:30PM	Soheil Memariansorkhabi
451	MWF	10:30AM	Youssef Hakiki	708	MWF	11:30AM	Nour Khoudari
501	MWF	11:30AM	Mathew George	709	MWF	11:30AM	Ying Zhang
502	MWF	3:30PM	Kuan-Ting Yeh	710	MWF	12:30PM	Ying Zhang

3. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices.
4. When time is called, all students must put down their writing instruments immediately. You may remain in your seat while your instructor will collect the exam booklets.
5. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above instructions regarding academic dishonesty:

STUDENT NAME ______________________________

STUDENT SIGNATURE ______________________________

STUDENT PUID ______________________________

SECTION NUMBER ______________________________

1. (10 points) Let

$$A = \begin{bmatrix} 1 & -1 & 3 & -4 & 2 \\ 1 & 0 & 1 & -1 & 1 \\ 0 & -1 & 2 & -3 & 1 \\ 1 & -2 & 5 & -7 & 3 \end{bmatrix}$$

Let a be the rank of A and b be the nullity of A, find $3a - 2b$.

A. -5

B. 0

C. 5

D. 10

E. 15

2. (10 points) Which of the following statements is always TRUE?

A. The nullity of a 4×7 matrix is at most 4.

B. The set of all the polynomials of degree (precisely) equal to 4 is a subspace of $\mathbb{P}_5$.

C. If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are linearly independent vectors in a vector space V, then $\mathbf{u} + \mathbf{v}, 2\mathbf{u} - 4\mathbf{w}, 3\mathbf{u} + 2\mathbf{v} - 2\mathbf{w}$ are also linearly independent vectors in V.

D. If B is an echelon form of a matrix A, then $\mathrm{Col}(A) = \mathrm{Col}(B)$.

E. If B is an echelon form of a matrix A, then $\mathrm{Col}(A^T) = \mathrm{Col}(B^T)$.

3. (10 points) Which of the following subsets of $M_{n\times n}$ is actually a subspace?

A. All $n \times n$ real diagonalizable matrices.

B. All $n \times n$ matrices A with each matrix entry a_{ij} being an integer.

C. All $n \times n$ matrices A with each matrix entry a_{ij} being non-negative.

D. All matrices A that can be written as $A = 3B - 2B^T$ for some $n \times n$ matrix B.

E. All $n \times n$ matrices whose rank is at most 1.

4. (10 points) Let $A = PDP^{-1}$ be a 3×3 diagonalizable matrix where P is an invertible matrix and D is a diagonal matrix with diagonal entries 3, 0, and -2. What are the eigenvalues of the matrix $A^3 - 2A$?

A. 6, 0, -4

B. 27, 0, -8

C. 21, 0, -4

D. 9, 0, -12

E. 3, 0, -2

5. (10 points) Consider the differential equation

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -1 & 2024 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

Then the origin is

A. a spiral point

B. a saddle point

C. an attractor

D. a repeller

E. none of the above

6. (10 points) Let $A = \begin{bmatrix} 3 & 1 \\ -2 & 5 \end{bmatrix}$, which of the following statements is not true?

A. $\begin{bmatrix} 1 \\ 1-i \end{bmatrix}$ is an eigenvector of the matrix A corresponding to eigenvalue $4-i$.

B. $\begin{bmatrix} 1+i \\ 2 \end{bmatrix}$ is an eigenvector of the matrix A corresponding to eigenvalue $4-i$.

C. $\begin{bmatrix} 1 \\ 1+i \end{bmatrix}$ is an eigenvector of the matrix A corresponding to eigenvalue $4+i$.

D. $\begin{bmatrix} -1+i \\ 2 \end{bmatrix}$ is an eigenvector of the matrix A corresponding to eigenvalue $4+i$.

E. The eigenvectors corresponding to the eigenvalues of matrix A are linearly independent.

7. (10 points) Let $\mathbb{P}_3$ be the set of all polynomials in one variable t of degree at most 3 and the zero polynomial. Denote

$$p_1(t) = 1 + t + t^2, \qquad p_2(t) = t - 1, \qquad p_3(t) = 1 + t.$$

Which of the following statements is false?

A. p_1, p_2 and p_3 are linearly independent.

B. $\{p_1, p_2, p_3\}$ is not a basis for $\mathbb{P}_3$.

C. $-3t^2 + 4t + 1$ is in the subspace spanned by $\{p_1, p_2, p_3\}$.

D. The kernel of the linear transformation given by $T(p) = p(0) - p(-1)$ contains a vector in the span of $\{p_1, p_2, p_3\}$.

E. $\text{Span}\{p_1, p_2\} = \text{Span}\{p_1, p_3\}$.

8. Let $T\colon \mathbb{P}_2 \longrightarrow \mathbb{P}_2$ be a linear transformation defined by

$$T(a_0 + a_1 t + a_2 t^2) = a_0 + (3a_0 - a_1)t + (4a_1 - a_2)t^2.$$

(2 points) (1) Find the image of $p(t) = 1 + 2t + 3t^2$.

(3 points) (2) Find the pre-image of $p(t) = 1 + 2t + 3t^2$.

(5 points) (3) Find the matrix $[T]_{\mathcal{B}}$ of the linear transformation T relative to the ordered standard basis $\mathcal{B} = \{1, t, t^2\}$ of $\mathbb{P}_2$.

9. (6 points) (1) Let $A = \begin{bmatrix} -3 & -3 & -1 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix}$, find all the eigenvalues and a basis for each eigenspace.

(4 points) (2) Find an invertible matrix P and a diagonal matrix D such that

$$\begin{bmatrix} -3 & -3 & -1 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix} = PDP^{-1}.$$

10. Given that vectors $\mathbf{v}_1 = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ are the eigenvectors of the matrix

$$A = \begin{bmatrix} 3 & 3 \\ 8 & 5 \end{bmatrix}.$$

(2 points) (1) Find their corresponding eigenvalues λ_1 and λ_2.

(2 points) (2) Find a general solution to the system of differential equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 8 & 5 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

(6 points) (3) Let $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ be a particular solution to the initial value problem

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 8 & 5 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 9 \\ -2 \end{bmatrix}.$$

Find $x(1) + y(1)$.

Please write your answers of the multiple choice questions in the following table.

Question	Answer
1. (10 points)	
2. (10 points)	
3. (10 points)	
4. (10 points)	
5. (10 points)	
6. (10 points)	
7. (10 points)	

Total Points: ________________________