MA 265, Spring 2022, Midterm 2 (GREEN)

INSTRUCTIONS:

1. **Write your answers of the seven multiple choice questions into the table on the last page. Show all your work on the questions and you may use the back of the test pages as scratch paper if needed.**

2. After you have finished the exam, hand in your test booklet to your instructor.

172	MWF	11:30AM	Zhang, Ying	253	TR	4:30PM	Kadattur, Shuddhodan
173	MWF	12:30PM	Zhang, Ying	264	TR	3:00PM	Kadattur, Shuddhodan
185	TR	12:00PM	Tsymbaliuk, Oleksandr	265	MWF	3:30PM	Nguyen, Thi-Phong
196	TR	1:30PM	Tsymbaliuk, Oleksandr	276	MWF	4:30PM	Nguyen, Thi-Phong
201	MWF	11:30AM	Debray, Arun	277	TR	12:00PM	Zhang, Qing
202	TR	12:00PM	Zhang, Zecheng	281	TR	1:30PM	Zhang, Qing
213	TR	4:30PM	Zhang, Zecheng	282	MWF	1:30PM	Tang, Shiang
214	MWF	4:30PM	Xu, Xuefeng	283	MWF	1:30PM	Zhang, Ying
225	MWF	3:30PM	Xu, Xuefeng	284	MWF	2:30PM	Debray, Arun
226	MWF	10:30AM	Yhee, Farrah	285	MWF	2:30PM	Tang, Shiang
237	MWF	11:30AM	Yhee, Farrah	287	TR	9:00AM	Rivera, Manuel
238	TR	9:00AM	Yang, Guang	288	MWF	10:30AM	Mohammad-Nezhad, Ali
240	TR	10:30AM	Yang, Guang	289	MWF	11:30AM	Mohammad-Nezhad, Ali
241	TR	3:00PM	Noack, Christian james	290	TR	10:30AM	Miller, Jeremy
252	TR	4:30PM	Noack, Christian james	291	TR	12:00PM	Ulrich, Bernd
				292	MWF	3:30PM	Heinzer, William

3. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices.

4. **Please remain seated during the last 10 minutes of the exam.** When time is called, all students must put down their writing instruments immediately. You may remain in your seat while your instructor will collect the exam booklets.

5. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above instructions regarding academic dishonesty:

STUDENT NAME ____________________

STUDENT SIGNATURE ____________________

STUDENT PUID ____________________

SECTION NUMBER ____________________

1. (10 points) Which of the following statements is not always true?

A. If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are linearly independent vectors in a vector space, then $\mathbf{u}+\mathbf{v}$, $\mathbf{v}+\mathbf{w}$, and $\mathbf{w}$ are also linearly independent vectors in the vector space.

B. Every linearly independent set of vectors in $\mathbb{R}^n$ consists of at most n vectors.

C. Every spanning set of $\mathbb{R}^n$ contains a basis of $\mathbb{R}^n$.

D. If the nullity of a matrix A is zero, then linear system $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b}$.

E. Any integer between 0 and 4, inclusive, can be the rank of a 6×4 matrix.

2. (10 points) Let $\mathbb{M}_{3\times 3}$ be the vector space of all 3×3 matrices, and let H be its subspace consisting of all A satisfying

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} A = A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

What is the dimension of H?

A. 1

B. 2

C. 3

D. 4

E. 5

3. (10 points) For which number(s) a does the matrix $A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ -2 & -1 & 0 & 0 \\ 10 & -9 & 6 & a \\ 1 & 5 & a & 3 \end{bmatrix}$ have 2 as an eigenvalue?

A. $a = 3$ only

B. $a = 3$ and $a = -3$ only

C. $a = 2$ only

D. $a = 2$ and $a = 3$ only

E. $a = 2$ and $a = -2$ only

4. (10 points) Let A and B be similar $n \times n$ matrices with real entries. Which of the following statements must be TRUE?

(i) A and B have the same characteristic polynomial.

(ii) If the columns of A are linearly independent, then 0 is an eigenvalue of A.

(iii) If A is diagonalizable, then all the eigenvalues of A must be nonzero.

(iv) If $-\lambda$ is an eigenvalue of A, then λ^4 is an eigenvalue of B^4.

(v) If A is diagonalizable, then B is diagonalizable.

A. (i), (ii), (v)

B. (i), (iii), (iv), (v)

C. (iv), (v)

D. (i), (iv), (v)

E. (i), (iii), (iv)

5. (10 points) Let $\mathbb{P}_3$ denote the vector space of all polynomials of degree at most 3, which of the following subsets are subspaces of either $\mathbb{R}^3$ or $\mathbb{P}_3$?

(i) The set of all vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$ such that $x + 2y + 3z = 1$.

(ii) The set of all vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$ such that $10x - 2y = z$.

(iii) The set of all polynomials $p(t)$ in $\mathbb{P}_3$ such that the degree of $p(t)$ is 3.

(iv) The set of all polynomials $p(t)$ in $\mathbb{P}_3$ satisfying $p(2) = 0$.

A. (ii) and (iv) only

B. (i), (ii) and (iv) only

C. (ii) and (iii) only

D. (ii), (iii) and (iv) only

E. (iii) and (iv) only

6. (10 points) Consider the differential equation

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

Then the origin is

A. an attractor

B. a repeller

C. a saddle point

D. a spiral point

E. none of the above

7. (10 points) A real 2×2 matrix A has an eigenvalue $\lambda_1 = 1 + i$ with corresponding eigenvector $\mathbf{v}_1 = \begin{bmatrix} 1 - 2i \\ 3 + 4i \end{bmatrix}$. Which of the following is the general **REAL** solution to the system of differential equations $\mathbf{x}'(t) = A\mathbf{x}(t)$?

A. $c_1 e^t \begin{bmatrix} \cos t + 2\sin t \\ 3\cos t - 4\sin t \end{bmatrix} + c_2 e^t \begin{bmatrix} \sin t - 2\cos t \\ 3\sin t + 4\cos t \end{bmatrix}$

B. $c_1 e^t \begin{bmatrix} \cos t - 2\sin t \\ 3\cos t + 4\sin t \end{bmatrix} + c_2 e^t \begin{bmatrix} \sin t + 2\cos t \\ 3\sin t - 4\cos t \end{bmatrix}$

C. $c_1 e^t \begin{bmatrix} \cos t + 2\sin t \\ 3\cos t + 4\sin t \end{bmatrix} + c_2 e^t \begin{bmatrix} \sin t - 2\cos t \\ 3\sin t - 4\cos t \end{bmatrix}$

D. $c_1 e^t \begin{bmatrix} -\cos t + 2\sin t \\ -3\cos t - 4\sin t \end{bmatrix} + c_2 e^t \begin{bmatrix} \sin t - 2\cos t \\ 3\sin t + 4\cos t \end{bmatrix}$

E. $c_1 e^t \begin{bmatrix} \cos t + 2\sin t \\ 3\cos t - 4\sin t \end{bmatrix} + c_2 e^t \begin{bmatrix} -\sin t - 2\cos t \\ -3\sin t + 4\cos t \end{bmatrix}$

8. Let $\mathbb{P}_2$ denote the vector space of all polynomials of degree at most 2 in the variable t, and let $\mathbb{M}_{2\times 2}$ denote the vector space of all 2×2 matrices. Consider a linear transformation:

$$T\colon \mathbb{P}_2 \to \mathbb{M}_{2\times 2} \quad \text{given by} \quad T(p(t)) = \begin{bmatrix} p(0) & p'(0) \\ p(1) & p'(1) \end{bmatrix}.$$

(1) (3 points) Find $T(at^2 + bt + c)$.

(2) (3 points) Find a polynomial $p(t)$ in $\mathbb{P}_2$ such that $T(p(t)) = \begin{bmatrix} 1 & 2 \\ 4 & 4 \end{bmatrix}$.

(3) (4 points) Find a basis for the range of T.

9. (6 points) (1) Find all the eigenvalues of matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 5 & 1 \\ -1 & -3 & 1 \end{bmatrix}$, and find a basis for the eigenspace corresponding to each of the eigenvalues.

(4 points) (2) Find an invertible matrix P and a diagonal matrix D such that

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 5 & 1 \\ -1 & -3 & 1 \end{bmatrix} = PDP^{-1}.$$

10. (4 points) (1) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix}.$$

(2 points) (2) Find a general solution to the system of differential equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

(4 points) (3) Let $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ be a particular solution to the initial value problem

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \qquad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Find $x(1) + y(1)$.

Please write your answers of the 7 multiple choice questions in the following table.

Question	Answer
1. (10 points)	
2. (10 points)	
3. (10 points)	
4. (10 points)	
5. (10 points)	
6. (10 points)	
7. (10 points)	

Total Points: ________________________