MA 265, Spring 2023, Midterm 2 (GREEN)

INSTRUCTIONS:

1. **Write your answers of the seven multiple choice questions into the table on the last page. Show all your work on the questions and you may use the back of the test pages as scratch paper if needed.**
2. After you have finished the exam, hand in your test booklet to your instructor.

172	MWF	9:30AM	Ying Zhang	265	TR	1:30PM	Shiang Tang
173	MWF	10:30AM	Ying Zhang	276	TR	3:00PM	Yiran Wang
185	MWF	10:30AM	Seongjun Choi	277	TR	1:30PM	Yiran Wang
196	MWF	9:30AM	Seongjun Choi	281	MWF	2:30PM	Siamak Yassemi
201	MWF	2:30PM	Jing Wang	282	MWF	1:30PM	Siamak Yassemi
202	TR	4:30PM	Takumi Murayama	283	MWF	11:30AM	Ying Zhang
213	MWF	4:30PM	Eric Samperton	284	MWF	11:30AM	Seongjun Choi
214	MWF	3:30PM	Eric Samperton	285	MWF	7:30AM	Luming Zhao
225	MWF	11:30AM	Farrah Yhee	287	MWF	8:30AM	Luming Zhao
226	MWF	10:30AM	Farrah Yhee	288	MWF	12:30PM	Ping Xu
237	TR	10:30AM	Ying Liang	289	MWF	1:30PM	Ping Xu
238	TR	12:00PM	Ying Liang	290	MWF	11:30AM	Ping Xu
240	MWF	2:30PM	Ayan Maiti	291	MWF	12:30PM	Yevgeniya Tarasova
241	MWF	1:30PM	Ayan Maiti	292	MWF	11:30AM	Yevgeniya Tarasova
252	TR	12:00PM	Vaibhav Pandey	293	MWF	11:30AM	William Heinzer
253	TR	1:30PM	Vaibhav Pandey	294	TR	1:30PM	Guang Lin
264	TR	3:00PM	Shiang Tang	295	MWF	3:30PM	William Heinzer

3. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices.
4. **Please remain seated during the last 10 minutes of the exam.** When time is called, all students must put down their writing instruments immediately.
5. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above instructions regarding academic dishonesty:

STUDENT NAME ______________________

STUDENT SIGNATURE ______________________

STUDENT PUID ______________________

SECTION NUMBER ______________________

1. (10 points) Let A be a 5×7 real matrix such that $rank(A) = 5$. Which of the following statements is true?

A. The dimension of the null space of A is equal to 0.

B. The columns of A are linearly independent.

C. The rank of A^T is equal to 7.

D. The rows of A are linearly independent.

E. The dimension of the row space of A is 2.

2. (10 points) Find the dimension of the subspace

$$H = \left\{ \begin{bmatrix} a - 2b + 9c + 5d + 4e \\ a - b + 6c + 5d - 3e \\ b - 3c + d - 9e \\ 5d - 10e \end{bmatrix} : a, b, c, d, e \in \mathbb{R} \right\}.$$

A. 1

B. 2

C. 3

D. 4

E. 5

3. (10 points) Consider the differential equation

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

Then the origin is

A. a saddle point

B. a spiral point

C. an attractor

D. a repeller

E. none of the above

4. (10 points) Which of the following statements are TRUE?

(i) If $n \times n$ matrices A and B have the same characteristic polynomial, then A must be similar to B.

(ii) If A is an $n \times n$ diagonalizable matrix, then A must have n distinct eigenvalues.

(iii) $n \times n$ matrix A and its transpose A^T must have the same eigenvalues.

(iv) There exists an $n \times n$ matrix whose eigenvectors span $\mathbb{R}^n$.

(v) If an $n \times n$ matrix A is invertible, then none of the eigenvalues of A can be zero.

A. (iii) and (v) only

B. (i), (ii) and (v) only

C. (ii) and (iii) only

D. (ii), (iv) and (v) only

E. (iii), (iv) and (v) only

5. (10 points) Which of the following sets describe a subspace of the vector space V?

A. The set of all vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ in $V = \mathbb{R}^2$ such that $\begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$.

B. The set of all vectors $\mathbf{v}$ in $V = \mathbb{R}^2$ such that $A\mathbf{v} = 3\mathbf{v}$ where $A = \begin{bmatrix} 3 & 0 \\ 2 & 3 \end{bmatrix}$.

C. The set of all vectors $\mathbf{v}$ in $V = \mathbb{R}^4$ such that $\mathbf{v} = \begin{bmatrix} x+1 \\ 1-x \\ 1 \\ 2x \end{bmatrix}, x \in \mathbb{R}$.

D. The set of all polynomials $p(t)$ in $V = \mathbb{P}_2 = \{\text{polynomials of degree at most } 2\}$ such that $p(1)p(2) = 0$.

E. The set of all $n \times n$ invertible matrices in $V = M_{n\times n}(\mathbb{R})$=$\{\text{all the } n \times n \text{ real matrices}\}$.

6. (10 points) Which of the following matrices are diagonalizable over the real numbers?

(i) $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} -3 & 1 \\ 0 & -3 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & 3 & 5 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ (iv) $\begin{bmatrix} 5 & -9 & 6 \\ 0 & 4 & -3 \\ 0 & 0 & 2 \end{bmatrix}$

A. (i) and (iv) only

B. (iii) and (iv) only

C. (i), (ii) and (iii) only

D. (i), (ii) and (iv) only

E. (i), (iii) and (iv) only

7. (10 points) Consider $\mathbf{x}' = A\mathbf{x}(t)$ with $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$. Which of the following is the general real solution?

A. $c_1 \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} e^t + c_2 \begin{bmatrix} \sin t \\ -\cos t \end{bmatrix} e^t$ with c_1, c_2 real numbers

B. $c_1 \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} e^t + c_2 \begin{bmatrix} -\cos t \\ \sin t \end{bmatrix} e^t$ with c_1, c_2 real numbers

C. $c_1 \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix} e^t$ with c_1, c_2 real numbers

D. $c_1 \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} -\cos t \\ \sin t \end{bmatrix} e^t$ with c_1, c_2 real numbers

E. $c_1 \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} e^t + c_2 \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} e^t$ with c_1, c_2 real numbers

8. Let $\mathbb{P}_2$ be the vector space consisting of all polynomials of degree at most 2. Note that $\mathbb{P}_2$ contains the zero polynomial. Consider the linear transformation $T : \mathbb{P}_2 \longrightarrow \mathbb{P}_2$ defined by

$$T(\mathbf{p}(t)) = \mathbf{p}(0) + \mathbf{p}(1)t + \mathbf{p}(2)t^2.$$

(4 points) (1) Find the image of $p(t) = t^2 - 1$ under the linear transformation T.

(6 points) (2) Find the matrix $[T]_{\mathcal{B}}$ for T relative to the ordered basis $\mathcal{B} = \{1, t, t^2\}$.

9. (6 points) (1) Find all the eigenvalues of matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 0 & -2 & 2 \end{bmatrix}$, and find a basis for the eigenspace corresponding to each of the eigenvalues.

(4 points) (2) Find an invertible matrix P and a diagonal matrix D such that

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 0 & -2 & 2 \end{bmatrix} = PDP^{-1}.$$

10. (4 points) (1) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} -4 & -5 \\ 2 & 3 \end{bmatrix}.$$

(2 points) (2) Find a general solution to the system of differential equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -4 & -5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

(4 points) (3) Let $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ be a particular solution to the initial value problem

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -4 & -5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \qquad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}.$$

Find $x(1) + y(1)$.

Please write your answers of the 7 multiple choice questions in the following table.

Question	Answer
1. (10 points)	
2. (10 points)	
3. (10 points)	
4. (10 points)	
5. (10 points)	
6. (10 points)	
7. (10 points)	

Total Points: ________________________