MATH 265 **Final Exam** Spring 2018

Name ______________________ **PUID** ______________________

Lecturer ______________________ **Section#** ______ **Class Time** __________

Exam Rules

1. You must use a #2 pencil on the scantron sheet (answer sheet).
2. Make sure that you have a **GREEN** scantron sheet. On the scantron, write **01** in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.
3. On the scantron, fill in your instructor's name and the **4 digit** section number (listed below).
4. Fill in your NAME and PUID and blacken in the appropriate spaces. BE SURE TO INCLUDE THE TWO LEADING ZEROS of your PUID.
5. You may not open the exam until instructed to do so.
6. You must obey the orders and requests made by all proctors, instructors, and lecturers.
7. Books, notes, calculators, or electronic devices are not allowed on the exam. You may not look at anybody else's test, and may not communicate with anybody else except, if you have a question, with the proctors.
8. There are 20 questions on this exam. Each question is worth 10 points. Each question has a unique answer/choice. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect.
9. There are 11 pages, including this cover page. You have 120 minutes to complete the exam.
10. After you finish the exam, hand in **both** your answer sheet **and** this test to your instructor.
11. After time is called, put down all writing instruments.
12. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Any act of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above.

Student Signature: ______________________

Here is a list of the sections:

Section #	Time	Instructor	Section #	Time	Instructor
0172	9:30am MWF	Chen, Ying	0173	10:30am MWF	Chen, Ying
0174	10:30am MWF	Ho, Meng-Che	0175	9:30am MWF	Ho, Meng-Che
0176	7:30am TR	Li, Shenghao	0177	9:00am TR	Li, Shenghao
0178	12:00pm TR	Liu, Yanghui	0179	1:30pm TR	Liu, Yanghui
0180	10:30am TR	Lorincz, Andras	0181	9:00am TR	Lorincz, Andras
0182	4:30pm TR	Madsen, Jeffrey	0183	3:00pm TR	Madsen, Jeffrey
0184	1:30pm MWF	Moon, Yong Suk	0185	2:30pm MWF	Moon, Yong Suk
0186	12:00pm TR	Patzt, Peter	0187	9:00am TR	Patzt, Peter
0188	12:30pm MWF	Urban, Roman	0189	11:30am MWF	Urban, Roman
0190	1:30pm TR	Watanabe, Tatsunari	0191	12:00pm TR	Watanabe, Tatsunari
0192	3:00pm TR	Xia, Jianlin	0193	10:30am TR	Xia, Jianlin
0194	1:30pm TR	Yang, Zhiguo	0195	3:00pm TR	Yang, Zhiguo
0196	11:30am MWF	Zhang, Ying	0197	12:30pm MWF	Zhang, Ying

1. Suppose a 3×3 real matrix $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ has determinant 5. What is the determinant of $B = \begin{bmatrix} 2a & 2c & 2b \\ 2d & 2f & 2e \\ 2g & 2i & 2h \end{bmatrix}$?

   A. 40

   B. 10

   C. -10

   D. 20

   E. -40

2. Let A be an $n \times n$ **singular** real matrix. Which of the following statements are always true?

   (i) $\det(\mathrm{A}) = 0$

   (ii) A is row equivalent to the identity matrix

   (iii) $A\mathbf{x} = \mathbf{0}$ must have nontrivial solutions

   (iv) $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b} \in \mathbb{R}^n$

   A. (i) only

   B. (i) and (iii) only

   C. (iii) only

   D. (ii) and (iv) only

   E. (i), (iii), and (iv)

3. Let $A = \begin{bmatrix} -6 & -2 & -8 \\ 6 & 0 & -4 \\ 2 & -2 & -6 \end{bmatrix}$. Suppose $A = S + K$, where S is symmetric, and K is skew symmetric (i.e., $K^T = -K$). Find K.

A. $K = \begin{bmatrix} 0 & 3 & -3 \\ -3 & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix}$

B. $K = \begin{bmatrix} 0 & -4 & -5 \\ 4 & 0 & -1 \\ 5 & 1 & 0 \end{bmatrix}$

C. $K = \begin{bmatrix} 0 & 5 & -2 \\ -5 & 0 & -7 \\ 2 & 7 & 0 \end{bmatrix}$

D. $K = \begin{bmatrix} -6 & 2 & -3 \\ 2 & 0 & -3 \\ -3 & -3 & -6 \end{bmatrix}$

E. $K = \begin{bmatrix} 8 & -7 & 5 \\ -7 & -5 & -8 \\ 5 & -8 & -8 \end{bmatrix}$

4. Let $A = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. The $(2, 1)$ entry (the entry in the second row and the first column) of A^{-1} is

A. $1/2$

B. 1

C. 0

D. $-1/2$

E. -1

5. For a real matrix A, suppose it has a row echelon form $\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Let V be the null space of A and $W = V^{\perp}$. What is the dimension of W?

A. 0

B. 1

C. 2

D. 3

E. 4

6. Let $A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ -1 & 1 & -1 & 0 \\ 3 & -1 & 7 & 0 \end{bmatrix}$. Which of following is a basis for the null space of A?

A. $\left\{ \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} -3 \\ -2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

E. None of the above

7. Compute the determinant of A, where

$$A = \begin{bmatrix} 1 & -3 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 & 4 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & -3 & 0 & -4 \\ 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -3 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix}^{-1}.$$

A. $\det(A) = -36$

B. $\det(A) = 5$

C. $\det(A) = 0$

D. $\det(A) = -13$

E. $\det(A) = 36$

8. Let $3a - 2b = 3$ and $2b - c = 4$. Find the value x determined by the linear system

$$\begin{bmatrix} a & b \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ 6 \end{bmatrix}.$$

A. $x = -4$

B. $x = 2$

C. $x = 3$

D. $x = 4$

E. $x = 6$

9. Consider real polynomials of degree less than or equal to 2, together with the zero polynomial. For which value of a does the polynomial $at^2 + t + 7$ belong to the span of the set

$$\{2t^2 + t + 5,\ t^2 - t - 3,\ 5t^2 - 2t - 4\}?$$

A. 1

B. 4

C. 5

D. 6

E. 3

10. Let M_{33} be the real vector space of all 3×3 real matrices. Which of the following sets is/are subspace(s) of M_{33}?

(i) The set of all 3×3 orthogonal real matrices

(ii) The set of all 3×3 symmetric real matrices

(iii) The set of all 3×3 singular real matrices

A. (ii) and (iii) only

B. (iii) only

C. (i) and (iii) only

D. (ii) only

E. (i) only

11. Let P_3 be the real vector space of polynomials of degree less than or equal to 3, together with the zero polynomial. Let W be the subspace of P_3 consisting of all the polynomials $p(t) \in P_3$ satisfying $p(0) = p(1)$. What is the dimension of W?

A. 0

B. 1

C. 2

D. 3

E. 4

12. Find all real number(s) a such that the following vectors form a basis for R^3 (the real vector space of all $n \times 1$ real vectors):

$$\begin{bmatrix} a \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} a \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

A. $a \neq 1$

B. $a \neq 2$

C. $a = 1$

D. $a = 2$

E. a can be any real number

13. Let A be a 5×7 real matrix such that the dimension of its column space is equal to 5. Which of the following statements is true?

A. The dimension of the null space of A is equal to 0

B. The columns of A are linearly independent

C. The rows of A are linearly independent

D. The rank of A^T is equal to 7

E. The dimension of the row space of A is 2

14. What are the resulting orthonormal vectors after applying the Gram-Schmidt process to the vectors

$$\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}?$$

A. $\begin{bmatrix} \frac{3}{\sqrt{14}} \\ \frac{1}{\sqrt{14}} \\ -\frac{2}{\sqrt{14}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{bmatrix}, \begin{bmatrix} \frac{2}{\sqrt{21}} \\ -\frac{4}{\sqrt{21}} \\ \frac{1}{\sqrt{21}} \end{bmatrix}$

B. $\begin{bmatrix} \frac{3}{\sqrt{14}} \\ \frac{1}{\sqrt{14}} \\ -\frac{2}{\sqrt{14}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{10}} \\ -\frac{3}{\sqrt{10}} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{3}{\sqrt{35}} \\ \frac{1}{\sqrt{35}} \\ \frac{5}{\sqrt{35}} \end{bmatrix}$

C. $\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} \\ -\frac{3}{2} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{6}{5} \\ \frac{2}{5} \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

E. $\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

15. Consider the inconsistent linear system $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} 0 & 0 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

The least squares solution $\hat{\mathbf{x}}$ of the linear system is

A. $\hat{\mathbf{x}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

B. $\hat{\mathbf{x}} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

C. $\hat{\mathbf{x}} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$

D. $\hat{\mathbf{x}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

E. None of the above

16. Let R^3 be the real vector space of all $n \times 1$ real vectors and M_{22} be the real vector space of all 2×2 real matrices. Let $L : R^3 \to M_{22}$ be a linear transformation that satisfies

$$L\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}, \quad L\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad L\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}.$$

What is $L\left(\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}\right)$ equal to?

A. $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

B. $\begin{bmatrix} 5 & 1 \\ 2 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

E. $\begin{bmatrix} -5 & -1 \\ -2 & -2 \end{bmatrix}$

17. Let A be an $n \times n$ matrix. Which of the following statements is/are always true?

(i) If A is diagonalizable, then A must have n distinct eigenvalues

(ii) If A is a matrix with all real entries, then all the eigenvalues of A must be real

(iii) If A is a symmetric real matrix, then it must be diagonalizable

A. (i) only

B. (iii) only

C. (i) and (ii) only

D. (i) and (iii) only

E. None of the statements is true

18. Let A be a 2×2 matrix. Suppose A has eigenvalues $\mathbf{i}$ and $-\mathbf{i}$ with the corresponding eigenvectors $\begin{bmatrix} 1 \\ \mathbf{i} \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -\mathbf{i} \end{bmatrix}$, respectively, where $\mathbf{i} = \sqrt{-1}$. Then A^{16} equals

A. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} \mathbf{i} & 0 \\ 0 & -\mathbf{i} \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1 \\ \mathbf{i} & -\mathbf{i} \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 \\ 1 & \mathbf{i} \end{bmatrix}$

E. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

19. For what value of a is the following linear system consistent? (Here, $\mathbf{i} = \sqrt{-1}$.)

$$\begin{aligned} -x + 2y &= a + 1 \\ 2x - 4y &= 2 + 2\mathbf{i} \end{aligned}$$

A. $a = 1 + \mathbf{i}$

B. $a = 2 + \mathbf{i}$

C. $a = -2 - \mathbf{i}$

D. $a = -2 + \mathbf{i}$

E. $a = -1 + 2\mathbf{i}$

20. Let $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ be the solution of the following initial value problem:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}.$$

Then $y(1)$ is equal to

A. $3e^6$

B. $-3e^6$

C. $3e^2$

D. $-3e^2$

E. $3e^6 - 3e^2$