MA26500 FINAL EXAM Spring 2022

GREEN - Test Version 01

NAME ______________________ INSTRUCTOR ______________________

1. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Use the back of the test pages for scratch paper. **PLEASE PUT YOUR SCANTRON UNDERNEATH YOUR QUESTION SHEETS WHEN YOU ARE NOT FILLING IN THE SCANTRON SHEET.**

2. Fill in your name and your instructor's name on the question sheets (above).

3. You must use a **#2 pencil** on the mark–sense sheet (answer sheet). Fill in the instructor's name and the course number(**MA265**), fill in the correct TEST/QUIZ NUMBER (**GREEN** is 01), your name, your section number (see below if you are not sure), and your 10-digit PUID(BE SURE TO INCLUDE THE TWO LEADING ZEROS of your PUID.) and shade them in the appropriate spaces. Sign the mark–sense sheet.

172	MWF	11:30AM	Zhang, Ying	253	TR	4:30PM	Kadattur, Shuddhodan
173	MWF	12:30PM	Zhang, Ying	264	TR	3:00PM	Kadattur, Shuddhodan
185	TR	12:00PM	Tsymbaliuk, Oleksandr	265	MWF	3:30PM	Nguyen, Thi-Phong
196	TR	1:30PM	Tsymbaliuk, Oleksandr	276	MWF	4:30PM	Nguyen, Thi-Phong
201	MWF	11:30AM	Debray, Arun	277	TR	12:00PM	Zhang, Qing
202	TR	12:00PM	Zhang, Zecheng	281	TR	1:30PM	Zhang, Qing
213	TR	4:30PM	Zhang, Zecheng	282	MWF	1:30PM	Tang, Shiang
214	MWF	4:30PM	Xu, Xuefeng	283	MWF	1:30PM	Zhang, Ying
225	MWF	3:30PM	Xu, Xuefeng	284	MWF	2:30PM	Debray, Arun
226	MWF	10:30AM	Yhee, Farrah	285	MWF	2:30PM	Tang, Shiang
237	MWF	11:30AM	Yhee, Farrah	287	TR	9:00AM	Rivera, Manuel
238	TR	9:00AM	Yang, Guang	288	MWF	10:30AM	Mohammad-Nezhad, Ali
240	TR	10:30AM	Yang, Guang	289	MWF	11:30AM	Mohammad-Nezhad, Ali
241	TR	3:00PM	Noack, Christian james	290	TR	10:30AM	Miller, Jeremy
252	TR	4:30PM	Noack, Christian james	291	TR	12:00PM	Ulrich, Bernd
				292	MWF	3:30PM	Heinzer, William

4. There are 25 questions, each is worth 8 points. Show your work on the question sheets. also **CIRCLE** your answer choice for each problem on the question sheets in case your scantron is lost. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

5. **Please remain seated during the last 10 minutes of the exam.** When time is called, all students must put down their writing instruments immediately.

6. **Turn in both the mark-sense sheets and the question sheets to your instructor when you are finished**.

1. For which value of a is the following system of equations in the variables x, y, and z consistent?

$$\begin{array}{llll} x & +2y & +3z & = 16 \\ 2x & & -2z & = 14 \\ 3x & +2y & +z & = 3a \end{array}$$

A. $a = -10$

B. $a = 10$

C. $a = 2$

D. $a = -2$

E. $a = 0$

2. If the determinant of the matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is 1, find the determinant of $\begin{bmatrix} d & 2a & g+d \\ e & 2b & h+e \\ f & 2c & i+f \end{bmatrix}$.

A. -2

B. 2

C. 4

D. -4

E. 1

3. Compute the determinant of A, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -1 & 3 & 7 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}^{-1}$.

A. 20

B. 10

C. -20

D. -10

E. 30

4. Let A and B be row-equivalent $m \times n$ matrices. Let $\mathbf{b}$ be a vector in $\mathbb{R}^m$. Which of the following statements must be TRUE?

(i) $\text{Row}\, A = \text{Row}\, B$

(ii) If $\text{Nul}\, B = \{\mathbf{0}\}$, then the columns of A are linearly dependent.

(iii) A and B have the same **reduced** echelon form.

(iv) A solution to $A\mathbf{x} = \mathbf{b}$ is also a solution to $B\mathbf{x} = \mathbf{b}$.

(v) $\text{Col}\, A = \text{Col}\, B$.

A. (iii), (iv), (v)

B. (ii), (iii), (iv), (v)

C. (i), (iii), (v)

D. (i), (v)

E. (i), (iii)

5. Let T be the linear transformation with standard matrix $A = \begin{bmatrix} 0 & 4 & 3 & 2 & 1 \\ 0 & 0 & 5 & 6 & 7 \\ 0 & 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, and let S be the linear map with standard matrix A^T. Which of the following statements must be TRUE?

A. T is one-to-one.

B. S is one-to-one.

C. S is onto.

D. The nullity of A^T is 1.

E. The range of T is $\mathbb{R}^5$.

6. Consider the following 3×3 matrix:

$$A = \begin{bmatrix} 1 & 2 & 7 \\ 1 & 3 & 12 \\ 2 & 5 & 20 \end{bmatrix}$$

Find the trace of the inverse matrix $A^{-1} = [b_{ij}]$, that is, calculate the sum $b_{11} + b_{22} + b_{33}$.

A. 10

B. 6

C. 7

D. 8

E. 9

7. Consider the following 3×5 matrix:

$$A = \begin{bmatrix} 1 & 2 & 2 & 10 & 3 \\ 2 & 4 & 1 & 11 & 5 \\ 3 & 6 & 2 & 18 & 1 \end{bmatrix}$$

Which of the following statements is <u>FALSE</u>?

A. $\left\{ \begin{bmatrix} 3 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} \right\}$ is a basis for $\operatorname{Col} A$

B. $\left\{ \begin{bmatrix} -4 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 3 \\ -1 \\ 0 \end{bmatrix} \right\}$ is a basis for $\operatorname{Nul} A$

C. $\left\{ \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 3 \\ -2 \\ 0 \end{bmatrix} \right\}$ is a basis for $\operatorname{Nul} A$

D. $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 1 \\ -3 \\ 1 \\ 0 \end{bmatrix} \right\}$ is a basis for $\operatorname{Nul} A$

E. $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 3 \end{bmatrix} \right\}$ is a basis for $\operatorname{Col} A$

8. Which of the following sets are subspaces of the space V described?

(i) The set of all vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ in $V = \mathbb{R}^2$ such that $\begin{bmatrix} 1 & 1 \\ 7 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$.

(ii) The set of all vectors $\mathbf{v}$ in $V = \mathbb{R}^3$ such that $A\mathbf{v} = 3\mathbf{v}$ where $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & -4 \\ 0 & 0 & 3 \end{bmatrix}$.

(iii) The set of all vectors $\mathbf{v}$ in $V = \mathbb{R}^4$ such that $\mathbf{v} = \begin{bmatrix} a+b+3d \\ a+c+1 \\ d-c-1 \\ b+2c \end{bmatrix}$, $a, b, c, d \in \mathbb{R}$.

(iv) The set of all polynomials $p(t)$ in $V = \mathbb{P}_3 = \{\text{polynomials of degree at most } 3\}$ such that $p'(2) = 0$.

A. (ii), (iii), and (iv) only

B. (ii) only

C. (i) and (ii) only

D. (ii) and (iv) only

E. (i) and (iv) only

9. Define the linear transformation $T : \mathbb{P}_2 \to \mathbb{P}_3$

$$T(a_0 + a_1 t + a_2 t^2) = a_2 t^3.$$

Then Ker(T) has a basis

A. $\{1, t\}$

B. $\{t, t^2\}$

C. $\{1, t, t^2, t^3\}$

D. $\{1, t^2\}$

E. $\{t^2, t^3\}$

10. Let A be an $m \times n$ real matrix. Which of the following statements is always TRUE?

A. The column space of A^T is a subspace of $\mathbb{R}^m$.

B. If $m = n$ and A is invertible, then $\det(2A^{-1}A^T) = 2$.

C. If $m = n$ and $A^2 = I_n$, then $\det(A) = 1$.

D. If $A^T A = 0$, then $A = 0$.

E. The null space of A is a subspace of $\mathbb{R}^m$.

11. Let

$$S = \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \right\}$$

and let H be the subspace in $\mathbb{R}^3$ spanned by the vectors in S, i.e. $H = \text{Span}(S)$. Which of the following is a basis for H?

A. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \right\}$

12. Consider the two vectors $\mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ in $\mathbb{R}^3$. Which of the following vectors is NOT in the subspace spanned by $\mathbf{x}$ and $\mathbf{y}$?

A. $\begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 8 \\ 13 \\ 5 \end{bmatrix}$

C. $\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} 5 \\ 8 \\ 3 \end{bmatrix}$

E. $\begin{bmatrix} 8 \\ 15 \\ 7 \end{bmatrix}$

13. Let A be the 2×2 matrix associated to rotation by 2 radians **counter-clockwise**. Then the eigenvalues of A are:

A. $\cos(2) + i\sin(2),\ -\cos(2) + i\sin(2)$

B. $\sin(2) + i\cos(2),\ \sin(2) - i\cos(2)$

C. $\cos(2) + i\sin(2),\ \cos(2) - i\sin(2)$

D. $\cos(2) + \sin(2),\ -\cos(2) + \sin(2)$

E. $\cos(2) - i\sin(2),\ -\cos(2) + i\sin(2)$

14. Which of the following statements are TRUE?

(i) A square matrix A is diagonalizable if and only if its transpose (i.e, A^T) is diagonalizable.

(ii) If a 3×3 real matrix has two real eigenvalues then all three eigenvalues are real.

(iii) If a 3×3 real matrix has two distinct eigenvalues then all three eigenvalues are distinct.

(iv) Matrix $B = AA^T$ is diagonalizable for any real $m \times n$ matrix A.

A. (i), (iii) and (iv)

B. (ii), (iii) and (iv)

C. (i), (ii) and (iii)

D. (i), (ii) and (iv)

E. (i), (ii), (iii) and (iv)

15. Let $A = \begin{bmatrix} \mathbf{v_1} & \mathbf{v_2} & \mathbf{v_3} & \mathbf{v_4} & \mathbf{v_5} & \mathbf{v_6} \end{bmatrix}$ be a 4×6 matrix. Suppose $\mathbf{v_1} = 2\mathbf{v_3} + 4\mathbf{v_5}$ and $\mathbf{v_4} = 5\mathbf{v_2} + \mathbf{v_6}$. Which of the following must be TRUE?

(i) The reduced echelon form of A has at least four pivots.

(ii) $\begin{bmatrix} 1 & 0 & -2 & 0 & -4 & 0 \end{bmatrix}^T$ is a vector in $\operatorname{Nul} A$.

(iii) The set $S = \{\mathbf{v_2}, \mathbf{v_3}, \mathbf{v_5}, \mathbf{v_6}\}$ is linearly independent.

(iv) $\operatorname{Col} A = \operatorname{Span}\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_6}\}$.

(v) $\operatorname{nullity}(A) \geq 2$.

A. (ii), (iv), (v)

B. (i), (iii), (iv)

C. (i), (ii)

D. (i), (iii),(iv), (v)

E. All of the statements are true.

16. Which of the following matrices are diagonalizable over the real numbers?

(i) $\begin{bmatrix} 10 & 0 & -2 \\ 0 & -6 & 1 \\ -2 & 1 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 2 & 5 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 & -1 & 4 \\ 0 & 5 & 2 \\ 0 & 0 & -1 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix}$

A. (i) , (ii), (iii) and (iv)

B. (i), (ii) and (iii) only

C. (i), (iii) and (iv) only

D. (i) and (iii) only

E. (ii) and (iv) only

17. Let $\mathcal{M}_{2\times 2}$ be the vector space of 2×2 matrices, and consider a linear transformation $T : \mathcal{M}_{2\times 2} \to \mathcal{M}_{2\times 2}$ given by $T(X) = A^{-1}XB$, where

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

Then the dimension of the range of T is

A. 0

B. 1

C. 2

D. 4

E. 3

18. Let A be a 2×2 matrix with trace $tr(A) = -2$ and determinant $det(A) = 11$. Consider the dynamical system $\mathbf{x}'(t) = A\mathbf{x}(t)$. Then the origin is

A. an attractor

B. a saddle point

C. a spiral point

D. a repeller

E. None of the above

19. Let $A = \begin{bmatrix} 4 & 0 & 0 \\ 3 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$. Which matrix P satisfies $P^{-1}AP = D$?

A. $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix}$

B. $P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

C. $P = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 2 \\ 1 & 1 & -1 \end{bmatrix}$

D. $P = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & -2 \\ 1 & 1 & 1 \end{bmatrix}$

E. $P = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$

20. Assume that all vectors and subspaces are in $\mathbb{R}^n$. Which of the following statements is FALSE?

A. If $\mathbf{z}$ is orthogonal to $\mathbf{u_1}$ and $\mathbf{u_2}$, and if $W = \text{Span}\{\mathbf{u_1}, \mathbf{u_2}\}$, then $\mathbf{z}$ is in $W^\perp$.

B. If $\mathbf{y} = \mathbf{z_1} + \mathbf{z_2}$, where $\mathbf{z_1}$ is in a subspace W and $\mathbf{z_2}$ is in $W^\perp$, then $\text{proj}_W \mathbf{y} = \mathbf{z_1}$.

C. If $\mathbf{x}$ is not in a subspace W, then $\mathbf{x} - \text{proj}_W \mathbf{x}$ is not zero.

D. If the columns of an $m \times n$ matrix A are orthonormal, then the linear mapping $\mathbf{x} \mapsto A\mathbf{x}$ preserves length.

E. The best approximation to $\mathbf{y}$ by elements of a subspace W is the vector $\mathbf{y} - \text{proj}_W \mathbf{y}$

21. Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, and $\mathbf{x} = \begin{bmatrix} 4 \\ 7 \\ 1 \end{bmatrix}$. Then the closest point in $W = \text{span}\{\mathbf{u}_1, \mathbf{u}_2\}$ to $\mathbf{x}$ is

A. $\begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} 4 \\ 7 \\ 1 \end{bmatrix}$

E. $\begin{bmatrix} 4 \\ 7 \\ -1 \end{bmatrix}$

22. Find a least-squares solution for $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 2 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix}$.

A. $\begin{bmatrix} -2 \\ -1 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

E. $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

23. Consider the differential equation:

$$x'(t) = x(t) + 4y(t)$$

$$y'(t) = 2x(t) + 3y(t)$$

with initial conditions $x(0) = -1$ and $y(0) = 2$. Then $x(2) =$

A. $-10e^{-2} - e^{10}$

B. $5e^{-1} - e^{5}$

C. $-e^{-1} + 2e^{5}$

D. $-2e^{-2} + e^{10}$

E. $2e^{-2} - e^{10}$

24. Let $C[-1,1]$ be the vector space of all continuous real-valued functions defined on $[-1,1]$. Define the inner product on $C[-1,1]$ by

$$\langle f, g \rangle = \int_{-1}^{1} f(t)g(t)dt.$$

Let W be the subspace spanned by the polynomials 1 and t. Find the orthogonal projection of $20t^3 - 12t^2$ onto W with respect to above inner product on $C[-1,1]$.

A. $4 + 12t$

B. $-4 + 12t$

C. $4 - 12t$

D. $20t^3 + 12t^2$

E. $20t^3 - 12t^2$

25. Consider the basis S for $\mathbb{R}^3$ given by

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}.$$

If we apply the Gram–Schmidt process to S to obtain an orthonormal basis, we obtain

A. $\begin{bmatrix} 2/\sqrt{3} \\ 1/\sqrt{3} \\ 2/\sqrt{3} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{3} \\ 2/\sqrt{3} \\ -2/\sqrt{3} \end{bmatrix}, \begin{bmatrix} -2/\sqrt{3} \\ 2/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$

B. $\begin{bmatrix} 2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix}, \begin{bmatrix} -2/3 \\ 2/3 \\ 1/3 \end{bmatrix}$

C. $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$

D. $\begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}, \begin{bmatrix} 1/3 \\ -1/3 \\ 1/3 \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$

E. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$