MA26600 Exam 1

GREEN VERSION

NAME: _____

PUID (10 digits): _____

INSTRUCTOR: _____

SECTION/TIME: _____

1. Fill in your NAME, your PUID (10 digits), your INSTRUCTOR's name and SECTION number (or class meeting TIME) above. Please print legibly and use your name as it appears in the Purdue directory; please don't shorten or use a nickname.
2. Make sure you have all 8 pages of the exam book.
3. There are 10 questions, each worth 10 points.
4. Questions 1–7 are multiple-choice questions. Indicate your choice of an answer by **circling the letter** next to the choice like this:

   (D.) My choice of a correct answer.

   Show your work on the question sheets in the space provided after each problem. Although no partial credit will be given on the multiple choice questions, any disputes about grades or grading will be settled by examining your written work on the question sheets.
5. Questions 8–10 are handwritten problems. **Write the solutions of the handwritten problems clearly and explain all steps.** You can use the back of the test pages for the scratch paper but it will not be looked for grading.
6. **NO CALCULATORS, BOOKS, NOTES, PHONES, OR CAMERAS ARE ALLOWED**.

   Turn off or put away all electronic devices.

1. Which of the following statements is true about the equilibrium solutions of the autonomous equation

$$\frac{dy}{dx} = -y^3 + 6y^2 - 9y.$$

A. It has no stable critical point

B. $y = 3$ is the only stable critical point

C. $y = 0$ is the only stable critical point

D. both $y = 0$ and $y = 3$ are stable

E. $y = -3$ is the only stable critical point

2. A tank initially contains 80 gal of brine containing 40 lb of salt. Brine containing 2 lb of salt per gallon enters the tank at the rate of 2 gal/min and the perfectly mixed brine in the tank flows out at the rate of 4 gal/min. Let $x(t)$ be the amount of salt at a time t in minutes. Then, the initial value problem that $x(t)$ satisfies is

A. $x' = 8 - \dfrac{4x}{40 - t}, \quad x(0) = 80$

B. $x' = 4 - \dfrac{4x}{80 - 2t}, \quad x(0) = 40$

C. $x' = 4 - \dfrac{x}{80 - 2t}, \quad x(0) = 40$

D. $x' = 1 - \dfrac{3x}{60 - 2t}, \quad x(0) = 80$

E. $x' = 2 - \dfrac{4x}{80 - t}, \quad x(0) = 80$

3. Let $y(x)$ be the solution to the initial value problem

$$xy' = 3y + 2x^4, \quad y(1) = 0.$$

Then, $y(2)$ is

A. 8

B. 4

C. 20

D. 16

E. 32

4. Apply Euler' s method to the differential equation

$$\frac{dy}{dx} = y - 2x$$

with $y(0) = 4$ and step size $h = 1$ to find an approximation of $y(3)$.

A. 24

B. 22

C. 14

D. -6

E. 2

5. Solve a homogeneous equation by using a substitution $v = y/x$.

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}, \quad x > 0.$$

A. $x^2 + 3y^2 = Cx^5$

B. $x^2 + 3y^2 = Cx^2e^{3x}$

C. $1 + y^2 = Cx$

D. $x^2 + y^2 = Cx^3$

E. $1 + 3y^2 = Ce^{3x}$

6. If the Wronskian $W(f, g) = -3e^{4t}$ and $f(t) = 4e^{2t}$, then $g(t)$ could be

A. $\frac{3}{4}te^{2t}$

B. $12e^{2t}$

C. $-\frac{3}{4}te^{2t}$

D. $-\frac{3}{4}te^{4t}$

E. $-\frac{3}{2}e^{2t}$

7. Find the general solution of $y^{(4)} - 8y'' + 16y = 0$.

A. $y = \cos(2x)(C_1 + C_2x) + \sin(2x)(C_3 + C_4x)$

B. $y = C_1 + C_2x + C_3x^2 + C_4x^3$

C. $y = C_1 + C_2e^{4x} + C_3e^{-4x}$

D. $y = C_1 + C_2\cos(4x) + C_3\sin(4x)$

E. $y = e^{-2x}(C_1 + C_2x) + e^{2x}(C_3 + C_4x)$

8. Solve the initial value problem

$$y'' + 2y' - 15y = 0,$$
$$y(0) = 3, \quad y'(0) = -7.$$

9. Explain why the following equation is exact and find an implicit formula for the solution to the initial value problem

$$(2xy - y^3)dx + (6y + x^2 - 3xy^2)dy = 0, \quad y(1) = 2.$$

10. Find a linear homogeneous constant-coefficient differential equation with the general solution $y(x) = C_1 e^{2x} + C_2 \cos(2x) + C_3 \sin(2x)$.