# MA26600 Exam 1

GREEN VERSION

NAME: ______________________________

PUID (10 digits): ______________________________

INSTRUCTOR: ______________________________

SECTION/TIME: ______________________________

1. Fill in your NAME, your PUID (10 digits), your INSTRUCTOR's name and SECTION number (or class meeting TIME) above. Please print legibly and use your name as it appears in the Purdue directory; please don't shorten or use a nickname.
2. Make sure you have all 9 pages of the exam book.
3. There are 10 questions, each worth 10 points.
4. Questions 1–7 are multiple-choice questions. Indicate your choice of an answer by **circling the letter** next to the choice like this:

   D. My choice of a correct answer.

   **Show your work** on the question sheets in the space provided after each problem. Although no partial credit will be given on the multiple choice questions, any disputes about grades or grading will be settled by examining your written work on the question sheets.
5. Questions 8–10 are handwritten problems. **Write the solutions of the handwritten problems clearly and explain all steps.** You can use the back of the test pages for the scratch paper but it will not be looked for grading.
6. **NO CALCULATORS, BOOKS, NOTES, PHONES, OR CAMERAS ARE ALLOWED**.

   Turn off or put away all electronic devices.

**1.** What is the largest open interval in which the solution of the initial value problem

$$(x^2 - 4)\frac{dy}{dx} - \frac{y}{x} = \frac{e^x}{x-5}, \qquad y(1) = 3.$$

is guaranteed to exist by the existence and uniqueness theorem.

A. $(0, 5)$
B. $(0, 2)$
C. $(-2, 2)$
D. $(-2, 5)$
E. $(0, 3)$

**2.** A tank contains 130 Liters of water and 50 grams of sugar. Water containing a sugar concentration of $4e^{-t}$ g/L flows into the tank at a rate of 3 L/min, and the mixture in the tank flows out at a rate of 4 L/min. Let $Q(t)$ be the amount of sugar (in grams) in the tank at time $t$ (in minutes). Which differential equation does $Q(t)$ satisfy?

A. $\dfrac{dQ}{dt} = 12e^{-t} - \dfrac{4Q}{130 - t}$
B. $\dfrac{dQ}{dt} = 3e^{-t} - \dfrac{4Q}{130 - t}$
C. $\dfrac{dQ}{dt} = 12e^{-t} - \dfrac{4Q}{130 + t}$
D. $\dfrac{dQ}{dt} = 12e^{-t} - \dfrac{4Q}{130}$
E. $\dfrac{dQ}{dt} = 4e^{-t} - \dfrac{Q}{50 + t}$

**3.** If $y$ is a solution to the the initial value problem

$$\frac{dy}{dx} - \frac{y}{x} = x^3, \quad y(1) = \frac{4}{3},$$

then $y(3) =?$

A. 20

B. 4

C. 27

D. 30

E. $\frac{8}{3}$

**4.** Use Euler's method with step size $h = 1$ to find the approximate value of $y(3)$, where $y(x)$ solves the initial value problem

$$y' = x + \frac{y}{2}, \quad y(0) = -8.$$

A. $-17$

B. $-22.5$

C. $-23.5$

D. $-24.5$

E. $-27$

**5.** Find the general solution of a homogeneous equation using substitution $v = \dfrac{y}{x}$.

$$\frac{dy}{dx} = \frac{4x^2 + 3y^2}{2xy}$$

A. $y^2 + 2x^2 = Cx^2$

B. $y^2 + 2x^2 = Cx$

C. $2y + 4x^2 = Cx^4$

D. $4y^2 + 4x^2 = Cx^3$

E. $y^2 + 4x^2 = Cx^3$

**6.** If the Wronskian $W(f, g) = 6e^{4t}$ and $f(t) = 2e^{2t}$, then $g(t)$ could be

A. $3te^{2t}$

B. $-3e^{2t}$

C. $6te^{2t}$

D. $6e^{2t}$

E. $Ae^{4t}$ for some constant $A$

**7.** If $y(x)$ is the solution to the initial value problem $\begin{cases} y'' - 2y' - 3y = 0, \\ y(0) = 4,\ y'(0) = 4, \end{cases}$

find $\quad y(1) + y'(1)$.

A. $2e^3 + 2e^{-1}$

B. $8$

C. $8e^{-3}$

D. $8e^3$

E. $8e^3 + 4e^{-1}$

**8.** Find a homogeneous third-order, linear differential equation with constant coefficients whose general solution is $y(x) = c_1 + c_2 e^{2x} + c_3 x e^{2x}$.

**9.** Given the differential equation for a population $x(t)$:

$$\frac{dx}{dt} = x\,(8 - x) - 7$$

a) Calculate the critical points of this equation and draw the phase diagram. Classify the stability of the critical points.

b) If the initial population $x(0) = 3$, predict the asymptotic behavior of the solution in the limit $t \to \infty$. Give a reason for your answer.

**10.** Explain why the following equation is exact and find an implicit formula for the solution to the initial value problem

$$(2xy - y^3)dx + (6y + x^2 - 3xy^2)dy = 0, \quad y(1) = 2.$$