MA26600 Exam 1

GREEN VERSION

NAME: ______________________________

INSTRUCTOR: ______________________________

SECTION/TIME: ______________________________

1. Fill in your NAME, your PUID (10 digits), your INSTRUCTOR's name and SECTION number (or class meeting TIME) above. Please print legibly and use your name as it appears in the Purdue directory; please don't shorten or use a nickname.
2. Make sure you have all 8 pages of the exam book.
3. There are 10 questions, each worth 10 points.
4. Questions 1–7 are multiple-choice questions. Indicate your choice of an answer by **circling the letter** next to the choice like this:

   D. My choice of a correct answer.

   Show your work on the question sheets in the space provided after each problem. Although no partial credit will be given on the multiple choice questions, any disputes about grades or grading will be settled by examining your written work on the question sheets.
5. Questions 8–10 are handwritten problems. **Write the solutions of the handwritten problems clearly and explain all steps.** You can use the back of the test pages for the scratch paper but it will not be looked for grading.
6. **NO CALCULATORS, BOOKS, NOTES, PHONES, OR CAMERAS ARE ALLOWED**.

   Turn off or put away all electronic devices.

1. Consider the initial value problem

$$y' = \sqrt{x - y}, \quad y(2) = y_0.$$

Find all values of y_0 for which the existence and uniqueness theorem **cannot** be used to guarantee the existence of a unique solution in an open interval containing 2.

A. $y_0 = 2$

B. $y_0 \leq 2$

C. $y_0 \geq 2$

D. $y_0 > 2$

E. $y_0 < 2$

2. If $y(x)$ is a solution to the initial value problem

$$\frac{dy}{dx} + \frac{y}{2 + 2x} = 6, \quad y(0) = 2,$$

then $y(3) =?$

A. 12

B. -17

C. 28

D. 15

E. -23

3. For the initial value problem $y' = y + t^2$, $y(0) = 1$, if we choose to use the Euler method with $h = \frac{1}{2}$ to compute an approximate value of $y(1)$, then what will we obtain?

A. $\dfrac{23}{8}$

B. $\dfrac{19}{8}$

C. $\dfrac{15}{8}$

D. $\dfrac{17}{8}$

E. $\dfrac{21}{8}$

4. A tank initially contains 100 gal of brine containing 50 lb of salt. Brine containing 1 lb of salt per gallon enters the tank at a rate of 5 gal/min, and the well-mixed brine in the tank flows out at the same rate. How much salt will the tank contain after 20 min?

A. $50 - 5e^{-20}$

B. $50 + 50e^{-2}$

C. $100 - 100e^{-1}$

D. $100 + 100e^{-1}$

E. $100 - 50e^{-1}$

5. Consider a population $P(t)$ satisfying the the initial value problem for the logistic equation

$$\frac{dP}{dt} = 0.01P(100 - P), \quad P(0) = P_0.$$

Which of the following statements are correct?

(i) If $P_0 > 0$, then $\lim_{t\to\infty} P(t) = 100$.

(ii) If $P_0 \neq 100$, then $P(t) \neq 100$ for all $t > 0$.

(iii) If $0 < P_0 < 100$ then then we can find $t > 0$ such that $P(t) = 0$.

A. Only (i)

B. Only (ii)

C. (i), (ii), (iii)

D. (i), (ii)

E. (ii), (iii)

6. Which of the following forms a fundamental set of solutions to the homogeneous differential equation

$$y^{(4)} + 8y'' + 16y = 0?$$

A. $\{\cos 2t, t\sin 2t, t\cos 2t, \sin 2t\}$

B. $\{\cos 2t, \sin 2t, e^{2t}, e^{-2t}\}$

C. $\{e^{2t}, te^{2t}, e^{-2t}, te^{-2t}\}$

D. $\{e^{2t}, e^{-2t}\}$

E. $\{e^{2t}\cos 2t, e^{2t}\sin 2t, e^{-2t}\cos 2t, e^{-2t}\sin 2t\}$

7. Solve the initial value problem

$$y'' + 6y' + 34y = 0; \qquad y(0) = 4, \quad y'(0) = -10.$$

A. $y(t) = 4e^{-3t} + \frac{2}{5}e^{-3t}$

B. $y(t) = 4e^{-3t}\cos(5t) + \frac{2}{5}e^{-3t}\sin(5t)$

C. $y(t) = e^{-6t}\cos(10t) + \frac{2}{5}e^{-6t}\sin(10t)$

D. $y(t) = 4e^{-3t}\cos(5t)$

E. $y(t) = 4e^{-3t}\cos(\sqrt{34}t) + \frac{2}{\sqrt{34}}e^{-3t}\sin(\sqrt{34}t)$

8. Find the value of the parameter α for which the equation is exact and then find an implicit solution of the initial value problem

$$(\alpha xy + 5y)dx + (x^2 + 5x - \sin y)dy = 0, \quad y(1) = 0.$$

9. It is known that one of the solutions of the differential equation

$$x^2y'' - xy' - 15y = 0 \qquad (x > 0)$$

is $y_1(x) = x^5$. Use the method of reduction of order to find a second linearly independent solution $y_2(x)$. Recall that this method consists of substituting $y_2(x) = v(x)y_1(x)$ into the differential equation above and reducing it to a first order equation for v'.

10. Solve the initial value problem

$$x^2y' - y^2 - xy = 0, \quad y(1) = 2,$$

by writing the equation in the homogeneous form $y' = f(y/x)$ and using the substitution $v = y/x$.