MA26600 Exam 2

GREEN VERSION

NAME: ______________________________

PUID (10 digits): ______________________________

INSTRUCTOR: ______________________________

SECTION/TIME: ______________________________

1. Fill in your NAME, your PUID (10 digits), your INSTRUCTOR's name and SECTION number (or class meeting TIME) above. Please print legibly and use your name as it appears in the Purdue directory; please don't shorten or use a nickname.
2. Make sure you have all 9 pages of the exam book.
3. There are 10 questions, each worth 10 points.
4. Questions 1–7 are multiple-choice questions. Indicate your choice of an answer by **circling the letter** next to the choice like this:

   D. My choice of a correct answer.

   Show your work on the question sheets in the space provided after each problem. Although no partial credit will be given on the multiple choice questions, any disputes about grades or grading will be settled by examining your written work on the question sheets.
5. Questions 8–10 are handwritten problems. **Write the solutions of the handwritten problems clearly and explain all steps.** You can use the back of the test pages for the scratch paper but it will not be looked for grading.
6. **NO CALCULATORS, BOOKS, NOTES, PHONES, OR CAMERAS ARE ALLOWED**.

   Turn off or put away all electronic devices.

1. Determine the appropriate form for a particular solution $y_p(x)$ to the third-order differential equation

$$y^{(3)} + y'' - y' - y = \cos x + xe^{-x}.$$

You may want to use that $r^3 + r^2 - r - 1 = (r^2 - 1)(r + 1)$.

A. $y_p(x) = A\cos x + Bxe^{-x}$

B. $y_p(x) = x^2(A\cos x + B\sin x) + (Cx + D)e^{-x}$

C. $y_p(x) = A\cos x + B\sin x + (Cx + D)e^{-x}$

D. $y_p(x) = A\cos x + B\sin x + x^2(Cx + D)e^{-x}$

E. $y_p(x) = A\cos x + x(Bx + C)e^{-x}$

2. A damped forced oscillation $x(t)$ satisfies the differential equation

$$x'' + 2x' + 5x = \cos(2t).$$

The steady periodic solution $x_{sp}(t)$ can be written in the form

$$x_{sp}(t) = C\cos(\omega t - \alpha).$$

What are the values of C, ω and α?

A. $C = \dfrac{\sqrt{17}}{17}$, $\omega = 2$, $\alpha = \tan^{-1}(1/4)$

B. $C = \sqrt{17}$, $\omega = 2$, $\alpha = \tan^{-1}(2/\sqrt{17})$

C. $C = \dfrac{1}{17}$, $\omega = 1$, $\alpha = \tan^{-1}(\sqrt{17}/17)$

D. $C = \dfrac{5}{17}$, $\omega = 1$, $\alpha = -\tan^{-1}(\sqrt{17}/17)$

E. $C = \dfrac{\sqrt{17}}{17}$, $\omega = 2$, $\alpha = \tan^{-1}(4)$

3. Consider a mass-spring-dashpot system with a position function $x(t)$ satisfying

$$mx'' + cx' + kx = 0.$$

Recall that $p = \frac{c}{2m}$, $\omega_0^2 = \frac{k}{m}$, and $\omega_1^2 = \omega_0^2 - p^2$. Which of the following is the general solution to the equation above for the *critically damped* case $c^2 = 4km$?

A. $x(t) = e^{pt}\left(c_1 + c_2 t\right)$

B. $x(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

C. $x(t) = e^{-pt}\left(c_1 + c_2 t\right)$

D. $x(t) = e^{-pt}\left(A\cos\omega_1 t + B\sin\omega_1 t\right)$

E. $x(t) = C\cos(\omega t - \alpha)$

4. Select the system of first-order differential equations that is equivalent to

$$2x'' + 4x' - 10x = e^t.$$

A. $\begin{cases} x_1' = x_2 \\ x_2' = -4x_2 + 10x_1 \end{cases}$

B. $\begin{cases} x_1' = x_2 \\ x_2' = 4x_2 + 10x_1 + e^t \end{cases}$

C. $\begin{cases} x_1' = -2x_2 + 5x_1 \\ x_2' = -x_1 \end{cases}$

D. $\begin{cases} x_1' = x_2 \\ x_2' = -2x_2 + 5x_1 + \frac{1}{2}e^t \end{cases}$

E. $\begin{cases} x_1' = 2x_2 \\ x_2' = -4x_2 + 10x_1 + e^t \end{cases}$

5. Consider the system

$$x_1' = \alpha x_1 + \beta x_2$$
$$x_2' = \alpha x_1 + \alpha x_2,$$

where α and β are constants. For what values of α and β is the origin a spiral sink?

A. $\alpha = 0$, $\beta > 0$

B. $\alpha < 0$, $\beta < 0$

C. $\alpha < 0$, $\beta > 0$

D. $\alpha < 0$, $\beta = 0$

E. $\alpha > 0$, $\beta < 0$

6. Consider the system

$$\mathbf{x}' = \begin{bmatrix} 3 & -4 \\ 1 & 7 \end{bmatrix} \mathbf{x}.$$

It is known that the system has one defective eigenvalue $\lambda = 5$. Find its general solution.

A. $\mathbf{x}(t) = c_1 \begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -2t+1 \\ t \end{bmatrix} e^{5t}$

B. $\mathbf{x}(t) = c_1 \begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{5t}$

C. $\mathbf{x}(t) = c_1 \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 4t+1 \\ -2t \end{bmatrix} e^{5t}$

D. $\mathbf{x}(t) = c_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 2t+1 \\ -t \end{bmatrix} e^{5t}$

E. $\mathbf{x}(t) = c_1 \begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -2t \\ t+1 \end{bmatrix} e^{5t}$

7. It is known that the general solution of the system

$$\mathbf{x}' = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \mathbf{x}$$

is given by

$$\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{t}.$$

Use the method of Undetermined Coefficients for systems to find the *form of a particular solution* to the nonhomogeneous system

$$\mathbf{x}' = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2t - 1 \\ 4e^t \end{bmatrix}.$$

A. $\mathbf{x}_p(t) = te^t\,\mathbf{a} + t\,\mathbf{b} + \mathbf{c}$

B. $\mathbf{x}_p(t) = te^t\,\mathbf{a} + e^t\,\mathbf{b} + t\,\mathbf{c} + \mathbf{d}$

C. $\mathbf{x}_p(t) = e^t\,\mathbf{a} + e^{3t}\,\mathbf{b} + t\,\mathbf{c} + \mathbf{d}$

D. $\mathbf{x}_p(t) = te^t\,\mathbf{a} + t^2\,\mathbf{b} + t\,\mathbf{c} + \mathbf{d}$

E. $\mathbf{x}_p(t) = t^2e^t\,\mathbf{a} + e^t\,\mathbf{b} + t\,\mathbf{c} + \mathbf{d}$

8. Using the method of Variation of Parameters, find a particular solution $y_p(x)$ of the differential equation

$$y'' - 4y' + 4y = \frac{2e^{2x}}{x}, \quad x > 0.$$

9. Consider the system

$$\begin{aligned} x_1' &= 2x_1 - 13x_2 \\ x_2' &= x_1 + 8x_2. \end{aligned}$$

(a) Find the real-valued general solution.

(b) Determine the type of the phase portrait at the origin.

10. Find the matrix exponential $e^{\mathbf{A}t}$ for the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ below:

$$\begin{aligned} x_1' &= 5x_1 - 4x_2 \\ x_2' &= 2x_1 - x_2. \end{aligned}$$