MA26600 Exam 2

GREEN VERSION

NAME: ______________________________

PUID (10 digits): ______________________________

INSTRUCTOR: ______________________________

SECTION/TIME: ______________________________

1. Fill in your NAME, your PUID (10 digits), your INSTRUCTOR's name and SECTION number (or class meeting TIME) above. Please print legibly and use your name as it appears in the Purdue directory; please don't shorten or use a nickname.
2. Make sure you have all 10 pages of the exam book.
3. There are 10 questions, each worth 10 points.
4. Questions 1–7 are multiple-choice questions. Indicate your choice of an answer by **circling the letter** next to the choice like this:

   D. My choice of a correct answer.

   Show your work on the question sheets in the space provided after each problem. Although no partial credit will be given on the multiple choice questions, any disputes about grades or grading will be settled by examining your written work on the question sheets.
5. Questions 8–10 are handwritten problems. **Write the solutions of the handwritten problems clearly and explain all steps.** You can use the back of the test pages for the scratch paper but it will not be looked for grading.
6. **NO CALCULATORS, BOOKS, NOTES, PHONES, OR CAMERAS ARE ALLOWED**.

   Turn off or put away all electronic devices.
7. Pull off the **table of Laplace transforms** on the last page of the exam for reference. Do not turn it in with your exam booklet at the end.

1. Using the method of undetermined coefficients, find the correct form of a particular solution y_p of

$$y''' - 4y' = t + 3\cos(t) + e^{-2t}.$$

A. $At + B + C\cos(t) + D\sin(t) + Ete^{-2t}$

B. $At^2 + Bt + C\cos(t) + D\sin(t) + Ee^{-2t}$

C. $At^2 + Bt + Ct\cos(t) + Dt\sin(t) + Ete^{-2t}$

D. $At + B + Ct\cos(t) + Dt\sin(t) + Ete^{-2t}$

E. $At^2 + Bt + C\cos(t) + D\sin(t) + Ete^{-2t}$

2. Select a differential equation which is equivalent to the following first order system

$$\begin{cases} x_1' = x_2, \\ x_2' = x_3, \\ x_3' = 2tx_1 - t^2x_2 - x_3 - 3e^t. \end{cases}$$

A. $x''' - 2tx'' + t^2x' + x = -3e^t$

B. $(x^3)' + x^3 + t^2x^2 - 2tx = -3e^t$

C. $x''' + x'' + t^2x' - 2tx = -3e^t$

D. $x^{(4)} - 2tx' + t^2x'' + x''' + 3e^tx = 0$

E. $x''' - x'' - t^2x' + 2tx - 3e^t = 0$

3. Find a general solution of the system of differential equations

$$\mathbf{x}' = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} \mathbf{x}.$$

A. $\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} t+1 \\ 1 \end{bmatrix} e^{3t}$

B. $\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} t+1 \\ t \end{bmatrix} e^{3t}$

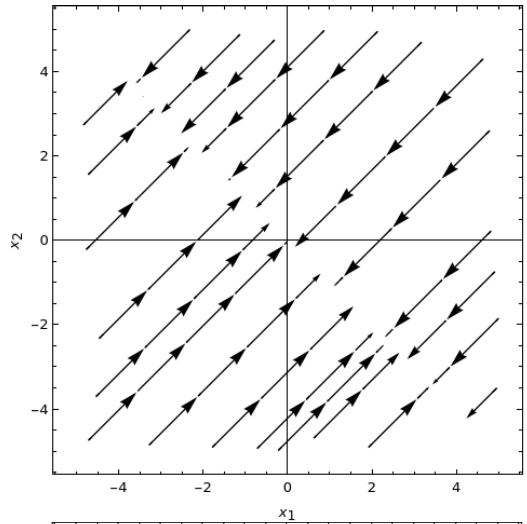
C. $\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} t-1 \\ t+1 \end{bmatrix} e^{3t}$

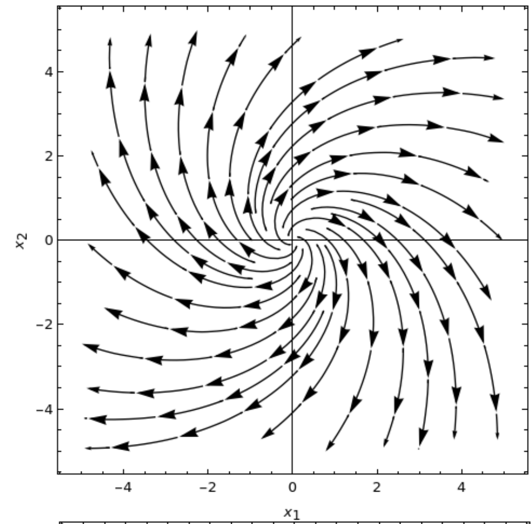
D. $\mathbf{x}(t) = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} -t+1 \\ t+1 \end{bmatrix} e^{3t}$

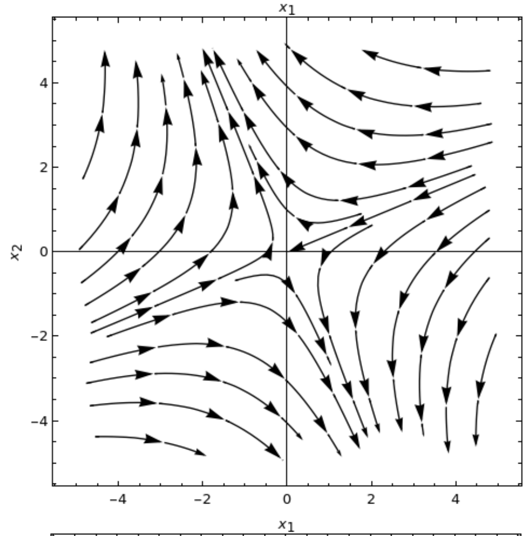
E. $\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} t \\ t+1 \end{bmatrix} e^{3t}$

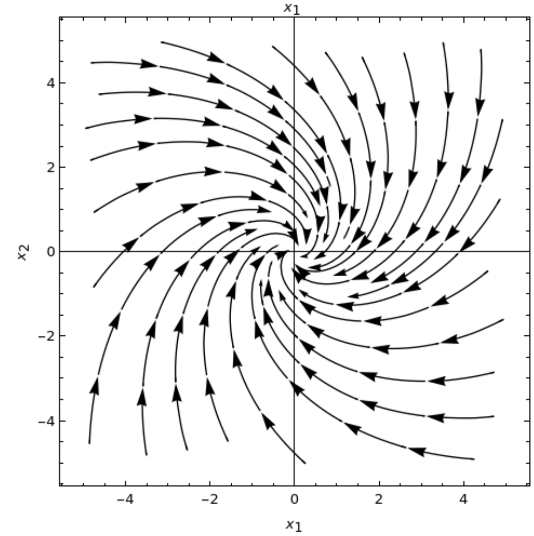
4. Identify the direction field of the following system of differential equations

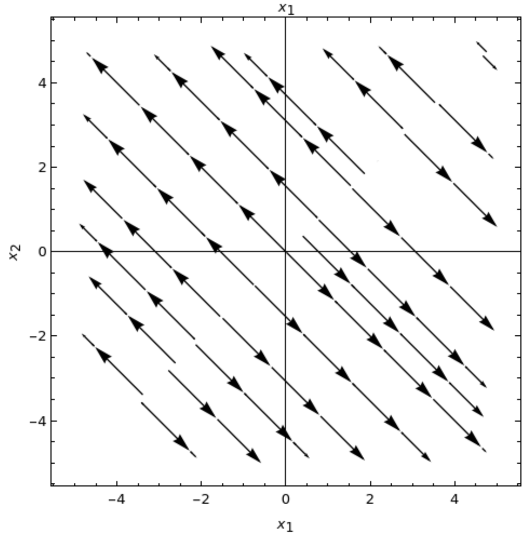
$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}.$$

A. 

B. 

C. 

D. 

E. 

5. Use the method of undetermined coefficients to find a particular solution $\mathbf{x}_p(t)$ of the nonhomogeneous system

$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^t,$$

given that $\mathbf{x}_c(t) = c_1 \begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t}$ is the general solution of the associated homogeneous system.

A. $\begin{bmatrix} -1/4 \\ 2 \end{bmatrix} e^t$

B. $\begin{bmatrix} 1/4 \\ 2 \end{bmatrix} e^t$

C. $\begin{bmatrix} 1/4 \\ -2 \end{bmatrix} e^t$

D. $\begin{bmatrix} -1/2 \\ 2 \end{bmatrix} e^t$

E. $\begin{bmatrix} 1/2 \\ -2 \end{bmatrix} e^t$

6. Find the matrix exponential $e^{\mathbf{A}t}$ for the matrix $\mathbf{A} = \begin{bmatrix} 3 & -6 \\ 0 & 3 \end{bmatrix}$.

A. $\begin{bmatrix} e^{3t} & -6te^{3t} \\ 0 & e^{3t} \end{bmatrix}$

B. $\begin{bmatrix} e^{3t} & e^{-6t} \\ 0 & e^{3t} \end{bmatrix}$

C. $\begin{bmatrix} 1+3t & 1-6t \\ 1 & 1+3t \end{bmatrix}$

D. $\begin{bmatrix} 1+e^{3t} & e^{-6t} \\ 0 & 1+e^{3t} \end{bmatrix}$

E. $\begin{bmatrix} 1+3t & -6t \\ 0 & 1+3t \end{bmatrix}$

7. Find the Laplace transform of the function $f(t) = (1+\cos t)^2$.

[*Hint.* You may want to use that $\cos^2 t = (1+\cos 2t)/2$.]

A. $\dfrac{1}{2s} + \dfrac{2s}{s^2+1} + \dfrac{s}{2(s^2+4)}$

B. $\dfrac{3}{2s} + \dfrac{2s}{s^2+1} + \dfrac{s}{2(s^2+4)}$

C. $\dfrac{1}{2s} + \dfrac{s}{s^2+1} + \dfrac{s}{2(s^2+4)}$

D. $\dfrac{3}{2s} + \dfrac{2s}{s^2+1} + \dfrac{s}{s^2+4}$

E. $\dfrac{3}{2s} + \dfrac{s}{s^2+1} + \dfrac{s}{s^2+4}$

8. Use variation of parameters to find a particular solution to

$$y'' - \frac{4}{x}\,y' + \frac{6}{x^2}\,y = 7, \qquad x > 0,$$

given that $y_1 = x^2$ and $y_2 = x^3$ are solutions of the corresponding homogeneous equation.

9. Solve the initial value problem

$$\mathbf{x}' = \begin{bmatrix} -1 & 3 \\ 3 & -1 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 5 \end{bmatrix}.$$

10. An undamped mass-spring system is described by the equation

$$x'' + x = 2\cos(\omega t)$$

(a) What value of ω would make the system experience resonance?

(b) Set ω to the value specified in part (a) and use the method of undetermined coefficients to find a particular solution of the system. Don't include complementary function(s) in your final answer.

Table of Laplace Transforms

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}, \quad s > 0$
2.	e^{at}	$\dfrac{1}{s-a}, \quad s > a$
3.	$t^n, \quad n = \text{positive integer}$	$\dfrac{n\,!}{s^{n+1}}, \quad s > 0$
4.	$t^p, \quad p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}, \quad s > 0$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}, \quad s > 0$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}, \quad s > \|a\|$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}, \quad s > \|a\|$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}, \quad s > a$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$
11.	$t^n e^{at}, \quad n = \text{positive integer}$	$\dfrac{n\,!}{(s-a)^{n+1}}, \quad s > a$
12.	$u(t-c)$	$\dfrac{e^{-cs}}{s}, \quad s > 0$
13.	$u(t-c)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(c\,t)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \quad c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$t^n f(t)$	$(-1)^n F^{(n)}(s)$