MA26600 Exam 2

GREEN VERSION

NAME: ______________________________

PUID (10 digits): ______________________________

INSTRUCTOR: ______________________________

SECTION/TIME: ______________________________

1. Fill in your NAME, your PUID (10 digits), your INSTRUCTOR's name and SECTION number (or class meeting TIME) above. Please print legibly and use your name as it appears in the Purdue directory; please don't shorten or use a nickname.

2. Make sure you have all 8 pages of the exam book.

3. There are 10 questions, each worth 10 points.

4. Questions 1–7 are multiple-choice questions. Indicate your choice of an answer by **circling the letter** next to the choice like this:

   (D.) My choice of a correct answer.

   Show your work on the question sheets in the space provided after each problem. Although no partial credit will be given on the multiple choice questions, any disputes about grades or grading will be settled by examining your written work on the question sheets.

5. Questions 8–10 are handwritten problems. **Write the solutions of the handwritten problems clearly and explain all steps.** You can use the back of the test pages for the scratch paper but it will not be looked for grading.

6. **NO CALCULATORS, BOOKS, NOTES, PHONES, OR CAMERAS ARE ALLOWED**.

   Turn off or put away all electronic devices.

1. The suitable form for a particular solution of the equation

$$y'' + 2y' + 5y = t^2 e^{-t} \sin 2t$$

if the method of Undetermined Coefficients is to be used is

A. $(A_2t^2 + A_1t + A_0)e^{-t} \sin 2t$

B. $t(A_2t^2 + A_1t + A_0)e^{-t} \sin 2t + t(B_2t^2 + B_1t + B_0)e^{-t} \cos 2t$

C. $At^2e^{-t} \sin 2t$

D. $t^2e^{-t}(A \sin 2t + B \cos 2t)$

E. $(A_2t^2 + A_1t + A_0)e^{-t} \sin 2t + (B_2t^2 + B_1t + B_0)e^{-t} \cos 2t$

2. A spring-mass system is governed by the following differential equation

$$3x'' + 48x = 7 \cos(2\omega t).$$

For what value(s) of ω will resonance occur?

A. 16

B. 4

C. 2

D. 8

E. No Value of ω

3. The trajectories of the system

$$\begin{cases} x' = y \\ y' = 9x \end{cases}$$

are [*Hint:* compute $x^2(t)$ and $y^2(t)$.]

A. Circles

B. Three straight lines

C. Infinitely many straight lines

D. Ellipses

E. Hyperbolas

4. Find the general solution of the following system of first order differential equations

$$\mathbf{x}' = \begin{bmatrix} 1 & 9 \\ 1 & 1 \end{bmatrix} \mathbf{x}.$$

A. $\mathbf{x} = C_1 e^{-2t} \begin{bmatrix} 3 \\ -1 \end{bmatrix} + C_2 e^{4t} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

B. $\mathbf{x} = C_1 e^{-2t} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + C_2 e^{4t} \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

C. $\mathbf{x} = C_1 e^{2t} \begin{bmatrix} 3 \\ -1 \end{bmatrix} + C_2 e^{-4t} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

D. $\mathbf{x} = C_1 e^{2t} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + C_2 e^{-4t} \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

E. $\mathbf{x} = (C_1 e^{-2t} + C_2 e^{4t}) \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

5. Consider a 2×2 matrix $\mathbf{A} = \begin{bmatrix} -1 & -2 \\ 5 & -3 \end{bmatrix}$. Then, a general solution of the linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ is

A. $\mathbf{x} = c_1 \left(\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cos 3t + \begin{bmatrix} 0 \\ -3 \end{bmatrix} \sin 3t \right) e^{-2t} + c_2 \left(\begin{bmatrix} 0 \\ -3 \end{bmatrix} \cos 3t + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \sin 3t \right) e^{-2t}$

B. $\mathbf{x} = c_1 \left(\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cos 3t + \begin{bmatrix} 0 \\ -3 \end{bmatrix} \sin 3t \right) e^{-2t} + c_2 \left(\begin{bmatrix} 0 \\ -3 \end{bmatrix} \cos 3t - \begin{bmatrix} 2 \\ 1 \end{bmatrix} \sin 3t \right) e^{-2t}$

C. $\mathbf{x} = c_1 \left(\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cos 3t + \begin{bmatrix} 0 \\ 3 \end{bmatrix} \sin 3t \right) e^{-2t} + c_2 \left(\begin{bmatrix} 0 \\ -3 \end{bmatrix} \cos 3t + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \sin 3t \right) e^{-2t}$

D. $\mathbf{x} = c_1 \left(\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cos 3t + \begin{bmatrix} 0 \\ 3 \end{bmatrix} \sin 3t \right) e^{-2t} + c_2 \left(\begin{bmatrix} 0 \\ 3 \end{bmatrix} \cos 3t + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \sin 3t \right) e^{-2t}$

E. $\mathbf{x} = c_1 \left(\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cos 3t + \begin{bmatrix} 0 \\ -3 \end{bmatrix} \sin 3t \right) e^{-t} + c_2 \left(\begin{bmatrix} 0 \\ 3 \end{bmatrix} \cos 3t - \begin{bmatrix} 2 \\ 1 \end{bmatrix} \sin 3t \right) e^{-3t}$

6. For the given values of the parameter α below, which value makes the origin a spiral source in the following system?

$$\mathbf{x}' = \begin{bmatrix} 1 & \alpha \\ 4 & 9 \end{bmatrix} \mathbf{x}$$

A. $\alpha = 0$

B. $\alpha = -4$

C. $\alpha = 4$

D. $\alpha = -10$

E. $\alpha = 10$

7. Find $e^{\mathbf{A}t}$ for the following matrix

$$\mathbf{A} = \begin{bmatrix} -4 & 0 \\ -1 & -4 \end{bmatrix}.$$

A. $e^{-4t} \begin{bmatrix} 4t & 0 \\ -t & 4t \end{bmatrix}$

B. $e^{-4t} \begin{bmatrix} 1 & 0 \\ -t & 1 \end{bmatrix}$

C. $e^{-4t} \begin{bmatrix} 1+4t & 0 \\ -t & 1+4t \end{bmatrix}$

D. $\begin{bmatrix} e^{-4t} & e^{0} \\ e^{-t} & e^{-4t} \end{bmatrix}$

E. $\begin{bmatrix} e^{-4t} & 0 \\ e^{-t} & e^{-4t} \end{bmatrix}$

8. Find the general solution of the following linear system

$$\mathbf{x}' = \begin{bmatrix} 5 & 4 \\ -1 & 1 \end{bmatrix} \mathbf{x}.$$

9. Use the method of Undetermined Coefficients to find the solution $y(x)$ of the initial value problem

$$y'' + 16y = 14 \sin 3x, \quad y(0) = 3,\ y'(0) = 2.$$

10. Use the method of Variation of Parameters to find the general solution of the nonhomogeneous equation

$$y'' + 6y' + 9y = \frac{e^{-3t}}{t^2} \qquad (t > 0).$$