MA 26600
FINAL EXAM INSTRUCTIONS
Dec 14, 2009

NAME ______________________ INSTRUCTOR ______________________

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. If the cover of your question booklet is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.

3. On the mark-sense sheet, fill in the instructor's name and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the SECTION NUMBER boxes with the division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number, ask your instructor.)

6. Sign the mark–sense sheet.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 23 questions, each worth an equal amount of points. Blacken in your choice of the correct answer in the spaces provided for questions 1–23. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.

9. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

10. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

11. A table of Laplace Transforms can be found on the last page of the question sheets.

1. Which of the following is the solution of

$$t^2y' + 2ty = \sin t, \; y(2\pi) = 0?$$

A. $\boxed{y(t) = \frac{1-\cos t}{t^2}}$

B. $y(t) = \frac{\sin t}{t^2}$

C. $y(t) = \frac{\cos t - 1}{t}$

D. $y(t) = \frac{\sin t}{t^3}$

E. $y(t) = \frac{1}{e^{t^2}} \int_{2\pi}^{t} e^{s^2} \sin s \, ds$

2. If $y = y(x)$ is the solution to

$$\frac{dy}{dx} = \frac{4xy}{2 + x^2}, \quad y(0) = 4,$$

then $y(1)$=

A. 2

B. 4

C. $\boxed{9}$

D. 1

E. 0

3. A huge tank initially contains 10 gallons (gal) of water with 6 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the well-stirred mixture is allowed to flow out of the tank at a rate of 2 gal/min. Which of the following is the amount of the salt in the tank after 10 min?

A. 10 lb

B. $\boxed{\text{19 lb}}$

C. 20 lb

D. 25 lb

E. 30 lb

4. Which of the following is the set of asymptotically stable equilibrium solution(s) of $\frac{dy}{dt} = y^2(y^2 - 4)$?

A. $y(t) = 0, 2$

B. $y(t) = 0, -2$

C. $y(t) = -2, 2$

D. $\boxed{y(t) = -2}$

E. $y(t) = 2$

5. Which of the following is an implicit solution to the initial value problem

$$(y\cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0, \quad y(-1) = 0?$$

A. $y\sin x - x^2e^y - y = -1$

B. $\boxed{y\sin x + x^2e^y - y = 1}$

C. $-\cos x + \frac{1}{3}x^3e^y - x = -\cos(-1) + \frac{2}{3}$

D. $y\cos x + x^2e^y - y = 1$

E. $y\sin x + x^2e^y + y = C$

6. If $y = y(x)$ is the solution to

$$y' = \frac{3y^2 + x^2}{2xy}, \quad y(1) = 1,$$

then $y(2) = ?$

A. $-2\sqrt{3}$

B. 1

C. $2\sqrt{2}$

D. $\boxed{2\sqrt{3}}$

E. 0

7. Given that the function $y_1 = t^2$ is a solution to the differential equation

$$t^2 y'' - t(t+4)y' + 2(t+3)y = 0,$$

choose a function y_2 from the list below so that the pair $\{y_1, y_2\}$ form a fundamental set of solutions to the differential equation above.

A. t^3

B. $t^2 \ln t$

C. $\boxed{t^2 e^t}$

D. $t^2 \sin t$

E. t

8. If the method of undetermined coefficients is to be used, the suitable form for a particular solution of the differential equation

$$y''' - 3y'' + 3y' - y = e^t + \sin t$$

is

A. $Ae^t + B \sin t$

B. $Ae^t + B \sin t + C \cos t$

C. $At\, e^t + B \sin t + C \cos t$

D. $At^2\, e^t + B \sin t + C \cos t$

E. $\boxed{At^3\, e^t + B \sin t + C \cos t}$

9. The general solution of the differential equation

$$y'' + 3y' + 2y = 4t$$

is

A. $e^{-t} + e^{-2t} + 4t$

B. $c_1e^t + c_2e^{2t} + 4t$

C. $c_1e^t + c_2e^{2t} + 2t - 3$

D. $\boxed{c_1e^{-t} + c_2e^{-2t} + 2t - 3}$

E. $c_1e^{-t} + c_2e^{-2t} + 4t$

10. The general solution of the differential equation

$$y^{(4)} - y = 0$$

is

A. c_1e^t

B. $c_1e^t + c_2e^{-t}$

C. $\boxed{c_1e^t + c_2e^{-t} + c_3\sin t + c_4\cos t}$

D. $c_1e^t + c_2e^{-t} + c_3e^t\sin t + c_4e^t\cos t$

E. $c_1e^t + c_2e^{-t} + c_3e^{-t}\sin t + c_4e^{-t}\cos t$

11. A mass weighing 4 lb stretches a spring 1 ft. the mass is attached to a viscous damper with a damping constant 0.5 lb-sec/ft. The mass is pulled down an additional 3 in, and then released. Let $u = u(t)$ denote the displacement of the mass from the equilibrium. (The gravity constant is $g = 32$ ft/sec^2.) Which of the following is satisfied by u?

A. $u'' + 4u' + 32u = 0,\ u(0) = 0,\ u'(0) = 0$

B. $4u'' + u' + 32u = 0,\ u(0) = 0.25,\ u'(0) = 0$

C. $4u'' + 0.5u' + 2u = 0,\ u(0) = 0,\ u'(0) = 0$

D. $\boxed{u'' + 4u' + 32u = 0,\ u(0) = 0.25,\ u'(0) = 0}$

E. $4u'' + 0.5u' + 2u = 0,\ u(0) = 0.25,\ u'(0) = 0$

12. One particular solution of the differential equation

$$y'' + y = \frac{1}{\sin t}, \quad 0 < t < \pi$$

is

A. $\frac{2}{\sin(t)}$

B. $t\cos t + (\ln|\sin t|)\ \sin t$

C. $\boxed{-t\cos t + (\ln|\sin t|)\ \sin t}$

D. $-t\sin t + (\ln|\sin t|)\ \cos t$

E. $\frac{1}{\cos(t)} + \frac{1}{\sin(t)}$

13. The inverse Laplace transform of $\dfrac{4s^2 - 4s + 12}{s(s^2+4)}$ is

A. $3 - \sin 2t + \cos 2t$

B. $3 + 2\sin 2t + \cos 2t$

C. $3 - 2\sin 2t$

D. $3 + 2\cos 2t$

E. $\boxed{3 - 2\sin 2t + \cos 2t}$

14. The solution of the initial value problem

$$y'' + 3y' + 2y = 2u_3(t); \quad y(0) = 0,\ y'(0) = 0$$

is

A. $\boxed{u_3(t)(1 - 2e^{-t+3} + e^{-2t+6})}$

B. $u_3(t)(2 - e^{-t+3} + 2e^{-2t+6})$

C. $u_3(t)(1 - 2e^{-t-3} + e^{-2t-6})$

D. $u_3(t)(2 + 2e^{-t+3} - e^{-2t+6})$

E. $u_3(t)(1 - e^{-t+3})$

15. The Laplace transform of

$$f(t) = \int_0^t (t-\tau)^3 \sin(2\tau) d\tau$$

is

A. $\dfrac{12}{s^6 + 4s^2}$

B. $\dfrac{12s}{s^6 + 4s^4}$

C. $\dfrac{12s}{s^6 + 4s^2}$

D. $\boxed{\dfrac{12}{s^6 + 4s^4}}$

E. $\dfrac{2}{s^6 + 4s^2}$

16. The Laplace transform of

$$f = \begin{cases} t^2 & 0 < t < 1 \\ 2t^2 & t \geq 1. \end{cases}$$

is

A. $\dfrac{2}{s^3} + \dfrac{2e^{-s}}{s^3}$

B. $\dfrac{2}{s^3} + e^{-s}\left(\dfrac{2}{s^3} + \dfrac{1}{s}\right)$

C. $\dfrac{2}{s^3} + e^{-s}\left(\dfrac{2}{s^3} + \dfrac{2}{s^2}\right)$

D. $\boxed{\dfrac{2}{s^3} + e^{-s}\left(\dfrac{2}{s^3} + \dfrac{2}{s^2} + \dfrac{1}{s}\right)}$

E. $\dfrac{2e^{-s}}{s^3}$

17. The solution of the initial value problem

$$y'' + 4y' + 4y = \delta(t-2), \qquad y(0) = 1,\ y'(0) = -2$$

is

A. $e^{2t} + u_2(t)te^{2t}$

B. $e^{2t} + u_2(t)(t-2)e^{2(t-2)}$

C. $2e^{-2t} + u_2(t)te^{-2t}$

D. $\boxed{e^{-2t} + u_2(t)(t-2)e^{-2(t-2)}}$

E. $e^{2t} + u_2(t)$

18. Consider the system

$$\mathbf{x}' = \begin{pmatrix} 0 & 4 \\ -1 & \alpha \end{pmatrix} \mathbf{x}.$$

For what value of α is the origin an asymptotically unstable node?

A. $0 < \alpha < 4$

B. $\alpha < -4$

C. $-4 < \alpha < 0$

D. $\boxed{\alpha > 4}$

E. all real α

19. In the phase portrait of the system $\mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} \mathbf{x}$, the origin is a

A. saddle point

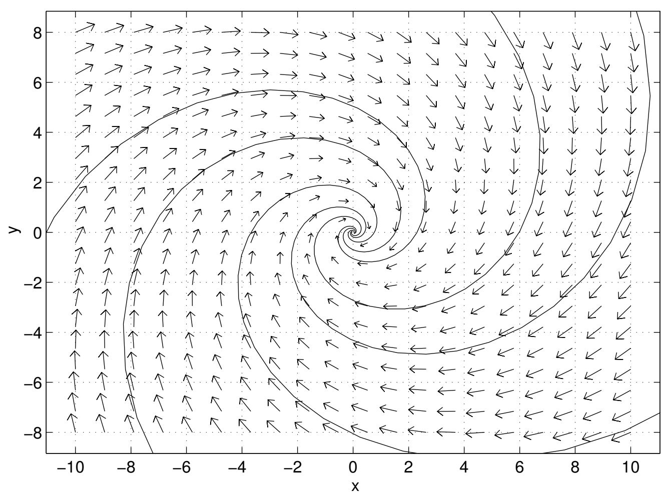
B. asymptotically stable node

C. asymptotically unstable node

D. asymptotically stable spiral point

E. asymptotically unstable spiral point

20. The phase portrait for a linar system of the form $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where $\mathbf{A}$ is a 2×2 matrix, is shown below. If r_1 and r_2 denote the eigenvalues of $\mathbf{A}$, then what can you conclude about r_1 and r_2 by examining the phase portrait?



A. r_1 and r_2 are distinct and positive

B. r_1 and r_2 are distinct and negative

C. r_1 and r_2 have opposite signs

D. r_1 and r_2 are complex and have positive real part

E. r_1 and r_2 are complex and have negative real part

21. The general solution of the system $\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} \mathbf{x}$ is

A. $\boxed{c_1 \begin{pmatrix} \cos t \\ 2\cos t + \sin t \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} \sin t \\ 2\sin t - \cos t \end{pmatrix} e^{-t}}$

B. $c_1 \begin{pmatrix} 1 \\ 5 \end{pmatrix} e^{-4t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t}$

C. $c_1 \begin{pmatrix} \cos t \\ 2\cos t - \sin t \end{pmatrix} e^{t} + c_2 \begin{pmatrix} -\sin t \\ -2\sin t - \cos t \end{pmatrix} e^{t}$

D. $c_1 \begin{pmatrix} 1 \\ -5 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t}$

E. $c_1 \begin{pmatrix} 2\cos t + \sin t \\ \cos t \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 2\sin t - \cos t \\ \sin t \end{pmatrix} e^{-t}$

22. The general solution of the system

$$\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \mathbf{x}$$

is

A. $c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} te^{2t}$

B. $\boxed{c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t} + c_2 \left[\begin{pmatrix} 1 \\ -1 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{2t} \right]}$

C. $c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t} + c_2 \left[\begin{pmatrix} 1 \\ -1 \end{pmatrix} te^{t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{t} \right]$

D. $c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{2t} + c_2 \left[\begin{pmatrix} -1 \\ 1 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{2t} \right]$

E. $c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + c_2 \left[\begin{pmatrix} -1 \\ 1 \end{pmatrix} te^{t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{t} \right]$

23. Which of the following is the general solution to the system

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} e^t$$

A. $c_1 \begin{pmatrix} -1 \\ 4 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$

B. $c_1 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t$

C. $\boxed{c_1 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t}$

D. $c_1 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^t$

E. $c_1 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t}$

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	t^n	$\dfrac{n!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
11.	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \quad c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^nf(t)$	$F^{(n)}(s)$