NAME ______________________________ INSTRUCTOR ______________________________

1. You must use a **#2 pencil** on the mark–sense sheet (answer sheet).
2. On the mark–sense sheet, fill in the **instructor's** name (if you do not know, write down the class meeting time and location) and the **course number** which is **MA266**.
3. Fill in your **NAME** and blacken in the appropriate spaces.
4. Fill in the **SECTION Number** boxes with section number of your class (see below if you are not sure) and blacken in the appropriate spaces.

0041	TR	12:00PM	Chi Li	0111	TR	9:00AM	Changyou Wang
0052	TR	10:30AM	Chi Li	0131	TR	12:00PM	Thomas Sinclair
0061	MWF	2:30PM	Heather Lee	0132	MWF	11:30AM	Chin-Yi Chan
0062	MWF	3:30PM	Heather Lee	0151	TR	3:30PM	Xu Zhang
0071	MWF	10:30AM	Daniel Phillips	0152	MWF	10:30AM	Chin-Yi Chan
0072	TR	7:30AM	Rachel Davis	0153	TR	10:30AM	Samy Tindel
0081	TR	9:00AM	Rachel Davis	0154	TR	12:00AM	Samy Tindel

5. Leave the TEST/QUIZ NUMBER blank.
6. Fill in the 10-DIGIT PURDUE ID and blacken in the appropriate spaces.
7. Sign the mark–sense sheet.
8. Fill in your name and your instructor's name on the question sheets (above).
9. There are 20 questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20 in the answer sheet. Do all your work on the question sheets. **Turn in both the mark-sense sheets and the question sheets when you are finished**.
10. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.
11. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.
12. **The Laplace transform table is provided at the end of the question sheets**.

1. Let $y(t)$ be a solution to

$$t^2 y' + 2ty = 2at, \quad \text{for} \quad t > 0.$$

If we know that

$$y(1) = 0, y'(1) = 2,$$

find the constant a.

A. 2

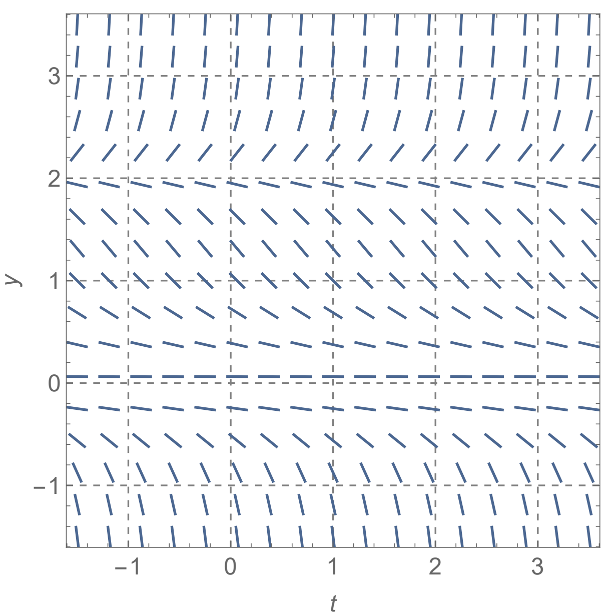
B. 1

C. 0

D. -1

E. -2

2. Identify the differential equation that produces the given direction field.



A. $y' = y(y-2)$

B. $y' = y(2-y)$

C. $y' = y^2(y-2)$

D. $y' = y(y-2)^2$

E. $y' = y^2(y-2)^2$

3. Which of the following functions can be a solution to

$$y' = \frac{x^2 + y^2}{xy}.$$

A. $y = x(C - 2\ln(x))$

B. $y = x(C + 2\ln(x))$

C. $y = x\sqrt{C - 2\ln x}$

D. $y = x\sqrt{C + 2\ln x}$

E. $y = Cx - 2\ln x$

4. Initially, a tank contains 400 L of water with 10 kg of salt in solution. Water containing 0.1kg of salt per liter (L) is entering at a rate of 1 L/min, and the mixture is allowed to flow out of the tank at a rate of 2 L/min. Let $Q(t)$ be the amount of salt at time t measured in kilograms. What is the right formulation of the differential equation for $Q(t)$?

A. $\frac{dQ}{dt} = 0.1 - \frac{2Q(t)}{400}$

B. $\frac{dQ}{dt} = \frac{0.1}{400} - \frac{2Q(t)}{400 - t}$

C. $\frac{dQ}{dt} = \frac{10}{400} - \frac{2Q(t)}{400 + t}$

D. $\frac{dQ}{dt} = 0.1 - \frac{2Q(t)}{400 + t}$

E. $\frac{dQ}{dt} = 0.1 - \frac{2Q(t)}{400 - t}$

5. Which one of the following statements is true about equilibrium solutions to $y' = 9y - y^3$?

A. $y = 3$ and $y = -3$ are unstable, $y = 0$ is asymptotically stable

B. $y = 0, y = 3, y = -3$ are all unstable

C. $y = 0, y = 3, y = -3$ are all asymptotically stable

D. $y = 3$ and $y = 0$ are asymptotically stable and $y = -3$ is unstable

E. $y = 3$ and $y = -3$ are asymptotically stable and $y = 0$ is unstable

6. The solution to the problem $2xy + 2xy^2 + 1 + (x^2 + 2x^2y + 2y)\,\dfrac{dy}{dx} = 0$ and $y(1) = 2$ is

A. $y^2 + x^2 + x^2y + x = 8$

B. $y^2x^2 + x^2y + y^2 = 10$

C. $x^2y + y^2x^2 + x = 7$

D. $x^2y + y^2x^2 + y^2 + x = 11$

E. $x^2y + y^2 + x = 7$

7. Use the Euler method to find the approximate value at $t = 1$ for $y' = 2x + y + 1,\ y(0) = 3$ with $h = 0.5$.

A. 5

B. 8.5

C. 3

D. 6.5

E. 4.5

8. On which interval is the initial value problem

$$\begin{cases} (5-t)y'' + (t-4)y' + 2y = \ln t \\ y(2) = 8 \end{cases}$$

guaranteed to have a unique solution?

A. $(-\infty, 5)$

B. $(0, \infty)$

C. $(0, 5)$

D. $(0, 4)$

E. $(-5, \infty)$

9. Find the general solution of the differential equation

$$y'' - 10y' + 27y = 0$$

A. $y = c_1e^{2t}\cos(\sqrt{5}t) + c_2e^{2t}\sin(\sqrt{5}t)$

B. $y = c_1e^{5t} + c_2e^{2t}$

C. $y = c_1e^{5t}\cos(\sqrt{2}t) + c_2e^{5t}\sin(\sqrt{2}t)$

D. $y = c_1e^{-5t} + c_2e^{-2t}$

E. $y = c_1e^{5t}\cos(\sqrt{5}t) + c_2e^{5t}\sin(\sqrt{5}t)$

10. The function $y_1 = e^x$ is a solution of

$$(x-1)y'' - 2xy' + (x+1)y = 0.$$

If we seek a second solution $y_2 = e^x v(x)$ by reduction of order, then $v(x) =$

A. $\frac{1}{3}(x-1)^{-3}$

B. $\frac{1}{2}(x-1)^2$

C. $(x-1)^2e^{-x}$

D. $\frac{1}{3}(x-1)^3$

E. $\frac{1}{3}(x-1)^3e^{-x}$

11. By the Method of Undetermined Coefficients, which of the following Y is the correct form of a particular solution to the equation

$$y^{(4)} - 3y^{(3)} + 2y^{(2)} = 4t - e^t + 3e^{3t}?$$

A. $Y = At + B + Ce^t + De^{3t}$

B. $Y = At^2 + Bt + Cte^t + De^{3t}$

C. $Y = At^3 + Bt^2 + Cte^t + De^{3t}$

D. $Y = At^3 + Bt^2 + Cte^t + Dte^{3t}$

E. None of the above.

12. The general solution to the homogeneous equation $t^2y'' + 7ty' + 5y = 0$ on the interval $0 < t < \infty$ is $y_c(t) = c_1t^{-1} + c_2t^{-5}$. A particular solution to the inhomogeneous equation $t^2y'' + 7ty' + 5y = t$ has the form $y_p(t) = u_1(t)t^{-1} + u_2(t)t^{-5}$. Which of the following are satisfied by u_1' and u_2'?

A. $u_1' = -t^5, \; u_2' = t$

B. $u_1' = \frac{t}{4}, \; u_2' = -\frac{t^5}{4}$

C. $u_1' = \frac{t^3}{4}, \; u_2' = -\frac{t^7}{4}$

D. $u_1' = -t^7, \; u_2' = t^3$

E. $u_1' = t, \; u_2' = t^5$

13. An undamped, free vibration $u'' + 9u = 0$ has initial conditions $u(0) = 4, u'(0) = 9$. The solution of this initial value problem can be written as $u = R\cos(\omega t - \delta)$. What are R and δ?

A. $R = 5$, $\delta = \frac{\pi}{2}$

B. $R = 6$, $\delta = \frac{\pi}{2}$

C. $R = 5$, $\delta = \tan^{-1}(\frac{3}{4})$

D. $R = 2\sqrt{5}$, $\delta = \tan^{-1}(\frac{3}{4})$

E. $R = 2\sqrt{5}$, $\delta = \tan^{1}(\frac{1}{2})$

14. Let $F(s) = \dfrac{6}{(s-3)^3}$ and $G(s) = \dfrac{5}{s^2+25}$. What is the inverse Laplace transform of the product of $F(s)G(s)$?

A. $3\displaystyle\int_0^t (t^2 - \tau)e^{3t-\tau}\sin(5\tau)d\tau$

B. $6\displaystyle\int_0^t (t^2 - \tau)e^{3(t-\tau)}\sin(5\tau)\,d\tau$

C. $3\displaystyle\int_0^t (t - \tau)^2 e^{3(t-\tau)}\sin(5\tau)\,d\tau$

D. $6\displaystyle\int_0^t (t - \tau)^2 e^{3(t-\tau)}\sin(5\tau)\,d\tau$

E. $3\displaystyle\int_0^t (t - \tau)^2 e^{3(t-\tau)}\sin(5(t-\tau))\,d\tau$

15. Which of the following functions has Laplace transform equal to

$$\frac{3s^2 + 4s - 1}{(s+1)(s^2 + 2s + 5)}?$$

A. $-\frac{1}{2}e^{-t} + \frac{7}{2}\cos(2t) + \frac{3}{4}\sin(2t)$

B. $-\frac{1}{2}e^{-t} + \frac{7}{2}e^{-t}\cos(2t) - e^{-t}\sin(2t)$

C. $-\frac{1}{2}e^{-t} + \frac{7}{2}e^{-t}\cos(2t) + \frac{3}{4}e^{-t}\sin(2t)$

D. $3e^{-t}\cos(2t) - e^{-t}\sin(2t)$

E. $-\frac{1}{2}e^{-t} + 3e^{-t}\cos(2t) - e^{-t}\sin(2t)$

16. Solve the initial value problem

$$y'' + 4y' + 5y = \delta\big(t - \frac{\pi}{4}\big), \;\; y(0) = 1, \;\; y'(0) = -2.$$

A. $e^{-2t}\cos t + u_{\frac{\pi}{4}}(t)e^{\frac{\pi}{2}-2t}\sin(t - \frac{\pi}{4})$

B. $e^{-2t}\cos t - u_{\frac{\pi}{4}}(t)e^{\frac{\pi}{2}-2t}\sin(t - \frac{\pi}{4})$

C. $e^{-2t}\cos t + u_{\frac{\pi}{4}}(t)e^{2t}\sin t$

D. $e^{2t}\cos t - u_{\frac{\pi}{4}}(t)e^{-2t}\sin t$

E. $e^{2t}\cos t + u_{\frac{\pi}{4}}(t)e^{-2t}\sin t$

17. The solution $x_2(t)$ determined by the initial value problem

$$\begin{cases} x_1' = x_2 \\ x_2' = x_1 \end{cases}$$

with initial conditions $x_1(0) = 1$, $x_2(0) = 2$ is given by

A. $x_2(t) = 2e^{-t} + 3e^t$

B. $x_2(t) = 2\sin t + \cos t$

C. $x_2(t) = \frac{1}{2}e^{-t} + \frac{3}{2}e^t$

D. $x_2(t) = 2\cos t - \sin t$

E. None of the above

18. In the phase portrait of the system

$$\mathbf{x}' = \begin{pmatrix} -3 & -5 \\ 3 & 1 \end{pmatrix} \mathbf{x},$$

the origin is a

A. asymptotically stable spiral point

B. saddle point

C. asymptotically stable node

D. asymptotically unstable node

E. asymptotically unstable spiral point

19. Find the solution of the following initial value problem

$$\mathbf{x}' = \begin{pmatrix} 3 & 6 \\ -1 & -2 \end{pmatrix} \mathbf{x}. \qquad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

A. $\mathbf{x}(t) = 4\begin{pmatrix} -2 \\ 1 \end{pmatrix} - 3\begin{pmatrix} -3 \\ 1 \end{pmatrix} e^t$

B. $\mathbf{x}(t) = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^t$

C. $\mathbf{x}(t) = -\frac{5}{7}\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t} + \frac{2}{7}\begin{pmatrix} 6 \\ 1 \end{pmatrix} e^{4t}$

D. $\mathbf{x}(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t} + \begin{pmatrix} 6 \\ 1 \end{pmatrix} e^{4t}$

E. $\mathbf{x}(t) = c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t} + c_2\begin{pmatrix} 6 \\ 1 \end{pmatrix} e^{4t}$

20. Consider the differential system:

$$\mathbf{x}' = \begin{pmatrix} -2 & 1 \\ -1 & -4 \end{pmatrix} \mathbf{x}.$$

A set of fundamental solutions to this system is given by

A. $\mathbf{x}^{(1)} = e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbf{x}^{(2)} = te^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + e^{-3t}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

B. $\mathbf{x}^{(1)} = e^{3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbf{x}^{(2)} = te^{3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + e^{3t}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

C. $\mathbf{x}^{(1)} = e^{-3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\mathbf{x}^{(2)} = te^{-3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + e^{-3t}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

D. $\mathbf{x}^{(1)} = e^{t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{x}^{(2)} = te^{t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + e^{t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

E. $\mathbf{x}^{(1)} = e^{3t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{x}^{(2)} = te^{3t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + e^{3t}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$