VERSION 01

MA26600 FINAL EXAM INSTRUCTIONS Spring 2015

NAME ______________________________ INSTRUCTOR ______________________________

1. You must use a **#2 pencil** on the mark–sense sheet (answer sheet).
2. On the mark–sense sheet, fill in the **instructor's** name (if you do not know, write down the class meeting time and location) and the **course number** which is **MA266**.
3. Fill in your **NAME** and blacken in the appropriate spaces.
4. Fill in the **SECTION Number** boxes with section number of your class (see below if you are not sure) and blacken in the appropriate spaces.

0031	MWF	11:30AM	Yang Yang	0081	MWF	01:30PM	Ying Chen
0041	MWF	12:30PM	Yih Sung	0091	TR	03:00PM	Guang Lin
0051	MWF	10:30AM	Johnny Brown	0111	MWF	12:30PM	Ying Chen
0052	TR	04:30PM	Guang Lin	0113	MWF	02:30PM	Moongyu Park
0053	TR	09:00AM	Kiril Datchev	0121	MWF	11:30AM	Moongyu Park
0061	MWF	10:30AM	Yang Yang	0122	MWF	01:30PM	Yih Sung
0062	MWF	09:30AM	Sai Kee Yeung	0141	MWF	09:30AM	Daniel Phillips
0063	TR	01:30PM	Changyou Wang	0151	TR	09:00AM	Changyou Wang

5. Enter 01 in the TEST/QUIZ NUMBER blank.
6. Fill in the 10-DIGIT PURDUE ID and blacken in the appropriate spaces.
7. Sign the mark–sense sheet.
8. Fill in your name and your instructor's name on the question sheets (above).
9. There are 20 questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20 in the answer sheet. Do all your work on the question sheets. **Turn in both the mark-sense sheets and the question sheets when you are finished**.
10. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.
11. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.
12. **The Laplace transform table is provided at the end of the question sheets**.
13. **Do not cheat.** Anyone caught cheating will lose their exam and be reported to the Dean of Students.

1. Let $y(t)$ be the solution to

$$ty' + 2y = 4t^2 \text{ for } t > 0,$$
$$y(1) = 5.$$

Find $y(2)$.

A. 1

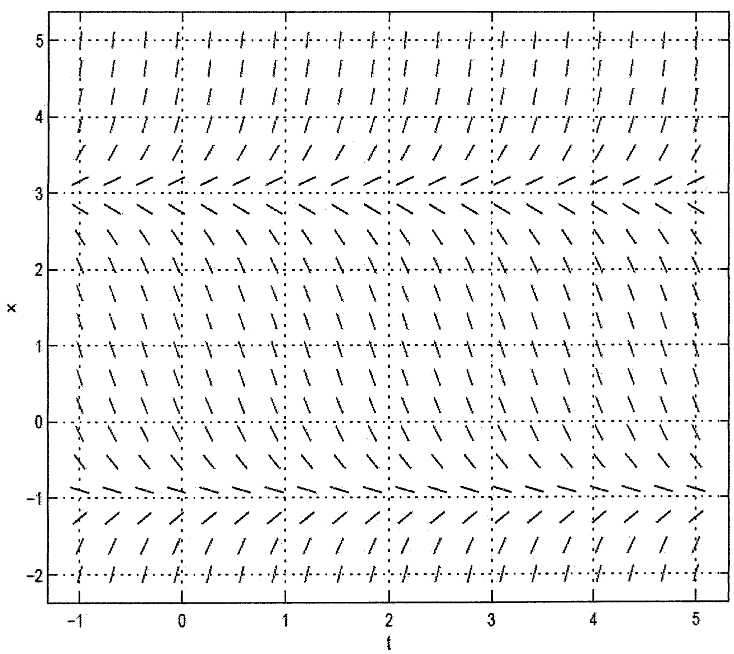
B. $\frac{7}{4}$

C. $\frac{5}{2}$

D. 4

E. 5

2. Identify the differential equation which produces the following direction field.



A. $\dfrac{dy}{dt} = (y+1)(y-3)$

B. $\dfrac{dy}{dt} = (y+1)(3-y)$

C. $\dfrac{dy}{dt} = (y-1)(y-3)$

D. $\dfrac{dy}{dt} = (y-1)(3-y)$

E. $\dfrac{dy}{dt} = y - 3$

3. Find the solution $y(x)$ to

$$\frac{dy}{dx} = e^{-\frac{y}{x}} + \frac{y}{x},$$
$$y(e) = 0.$$

A. $y(x) = \ln(\ln(x))$
B. $y(x) = x\ln(\ln(x))$
C. $y(x) = \ln(\ln(x)+1) - \ln(2)$
D. $y(x) = x\ln(x) - e$
E. $y(x) = x\ln(x) - x$

4. Consider a pond that initially contains 10 million gal of water. Water containing a polluted chemical flows into the pond at the rate of 6 million gal/yr, and the mixture in the pond flows out at the rate of 5 million gal/yr. The concentration $\gamma(t)$ of chemical in the incoming water varies as $\gamma(t) = 2 + \sin 2t$ grams/gal. Let $Q(t)$ be the amount of chemical at time t measured by millions of grams. Then what is the right formulation of the differential equation?

A. $Q' = 6(2+\sin 2t) - \frac{Q}{(10-t)}.$

B. $Q' = 6(2+2\cos 2t) - \frac{5Q}{(10-t)}.$

C. $Q' = 6 \cdot 10^6(2+\sin 2t) - \frac{Q}{(10+t)}.$

D. $Q' = 6 \cdot 10^6(2+2\cos 2t) - \frac{Q}{10^7}.$

E. $Q' = 6(2+\sin 2t) - \frac{5Q}{(10+t)}.$

5. Find the solution for $\frac{1}{x}\frac{dy}{dx} = \frac{g(x)}{y}$ in implicit form, where $g(x)$ is a continuous function.

A. $y^2 - g(x)x^2 = c,$

B. $y - \int \frac{xg(x)}{y}\, dx = c.$

C. $y^2 - 2\int xg(x)\, dx = c.$

D. $y^2 - xg(x)\ln y = c,$

E. $y^2 - x^2 g(x) - \int g(x)\, dx = c.$

6. What is the largest open interval in which the solution of the initial value problem below is guaranteed to exist by the Existence and Uniqueness Theorem ?

$$\begin{cases} (t^2 + t - 2)\, y' + \frac{e^t}{\sin t}\, y = \frac{(t-4)}{(t-6)} \\ y'(2) = -1 \end{cases}$$

A. $(0, \pi)$

B. $(-2, \pi)$

C. $(0, 6)$

D. $(1, \pi)$

E. $(-\pi, \pi)$

7. Which statements below are **TRUE** for this autonomous equation $\dfrac{dy}{dt} = 8(y^2-4)^2(y^2+2y)$?

(I) There are precisely three equilibrium solutions.

(II) $y = 0$ is an asymptotically stable equilibrium solution.

(III) $y = -2$ is an asymptotically stable equilibrium solution.

A. Only **(II)**

B. Only **(I)** and **(III)**

C. Only **(II)** and **(III)**

D. Only **(I)** and **(II)**

E. All three are **TRUE**

8. Find an implicit solution of the initial value problem:

$$\begin{cases} (6x^2y^2 + 4e^x - 2y\sin 2x) + (4x^3y + \cos 2x)\dfrac{dy}{dx} = 0, \\ y(0) = 1 \end{cases}.$$

A. $2x^3y^2 - y\cos 2x + 4e^x = 3$

B. $x^3y^2 + y\cos 2x + 4ye^x = 0$

C. $2x^3y^2 + y\cos 2x + 4e^x = 5$

D. $x^3y^2 + y^2\cos 2x + 4e^x = 5$

E. $2x^3y^2 + y\cos 2x - 4ye^x = -3$

9. Find the general solutions to $y'' + 6y' + 10y = 0$

A. $c_1e^{-2t}\cos(t) + c_2e^{-2t}\sin(t)$
B. $c_1e^{-3t}\cos(t) + c_2e^{-3t}\sin(t)$
C. $c_1e^{-3t}\cos(2t) + c_2e^{-3t}\sin(2t)$
D. $c_1e^{2t}\cos(t) + c_2e^{2t}\sin(t)$
E. $c_1e^{3t}\cos(t) + c_2e^{3t}\sin(t)$

10. Solve the initial value problem

$$y'' - 4y = \cos(t), \quad y(0) = 0, \ y'(0) = 0,$$

A. $\frac{1}{10}e^{2t} - \frac{1}{10}e^{-2t} - \frac{1}{5}\cos(t)$
B. $\frac{1}{10}e^{2t} + \frac{1}{10}e^{-2t} - \frac{1}{5}\cos(t)$
C. $\frac{1}{10}e^{2t} - \frac{1}{10}e^{-2t} - \frac{1}{5}\sin(t)$
D. $\frac{1}{10}e^{2t} + \frac{1}{10}e^{-2t} - \frac{1}{5}\cos(t) - \frac{1}{5}\sin(t)$
E. $\frac{1}{5}e^{2t} + \frac{1}{5}e^{-2t} - \frac{2}{5}\cos(t)$

11. If the method of undetermined coefficients is to be used, find the suitable form for a particular solution of the differential equation

$$y'' + 2y' + y = t + e^{-t}.$$

A. $At + Be^{-t}$

B. $At + Bte^{-t}$

C. $At + Bte^{-t} + Ct^2e^{-t}$

D. $At + B + Ct^2e^{-t}$

E. $At + B + Ct^3e^{-t}$

12. Given that a general solution of the homogeneous equation, $t^2y'' + 2ty' - 2y = 0$ on the domain $t > 0$ is $y = c_1t + c_2t^{-2}$. Using Variation of Parameters, a particular solution $u_1t + u_2t^{-2}$ of the equation $t^2y'' + 2ty' - 2y = 6t$ is found. Which of the following are satisfied by u_1' and u_2'?

A. $u_1' = 2t^3,\ u_2' = -6t^5.$

B. $u_1' = 3,\ u_2' = -t^2.$

C. $u_1' = t^2,\ u_2' = -3t^5.$

D. $u_1' = 5t^{-2},\ u_2' = -5t.$

E. $u_1' = 2t^{-1},\ u_2' = -2t^2.$

13. An undamped forced oscillation $u''+4u=0$ has initial conditions $u(0)=4, u'(0)=6$. The solution of the initial value problem can be written as $u = R\cos(\omega t - \delta)$. What are R and δ?

A. $R = 5,\ \delta = \frac{\pi}{4}$

B. $R = 4,\ \delta = \frac{\pi}{4}$

C. $R = 2\sqrt{13},\ \delta = \tan^{-1}(\frac{1}{2})$

D. $R = 5,\ \delta = \tan^{-1}(\frac{3}{4})$

E. $R = \sqrt{13},\ \delta = \tan^{-1}(\frac{1}{2})$

14. Find the general solution to

$$L[y] = y^{(4)} + 2y^{(3)} - 3y'' - 8y' - 4y = 0.$$

Note that $L[e^{rt}] = (r+1)^2(r^2-4)e^{rt}$.

A. $Ae^{-t} + Bte^{-t} + Ce^{-2t} + Dte^{2t}$

B. $Ate^{-t} + Bt^2e^{-t} + Ce^{-2t} + Dte^{-2t}$

C. $Ae^{-t} + Bte^{-t} + Ce^{-4t} + Dte^{-4t}$

D. $Ate^{-t} + Bt^2e^{-t} + Ce^{2t} + Dte^{-2t}$

E. $Ae^{-t} + Bte^{-t} + Ce^{2t} + De^{-2t}$

15. Let

$$f(t) = u_1(t)e^t - u_2(t)e^t.$$

If $F(s)$ is the Laplace transform of $f(t)$, what is $F(2)$?

A. $\dfrac{e^2 - e}{2}$

B. $\dfrac{e^{-2} - e^{-1}}{2}$

C. $e^{-2} - e^{-1}$

D. $e^{-1} - e^{-2}$

E. $e^2 - e$

16. Let $y(t)$ be the solution to the initial value problem

$$y'' + 2y' + y = \delta(t-3), \qquad y(0) = 0,\ y'(0) = 0.$$

What is $y(4)$?

A. $-e^2$

B. $-e$

C. e^{-1}

D. e^{-2}

E. e

17. If

$$F(s) = \mathfrak{L}\left\{ \int_0^t \sin(t-\tau)e^{2\tau}\,d\tau \right\}$$

find $F(3)$.

A. 1/10
B. 3/10
C. 5/10
D. 7/10
E. 9/10

18. In the phase portrait of the system

$$\mathbf{X}' = \begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix} \mathbf{X}$$

the origin is a

A. saddle point
B. asymptotically stable node
C. asymptotically unstable node
D. asymptotically stable spiral point
E. asymptotically unstable spiral point

19. Find the solution of the given initial value problem

$$\mathbf{X}' = \begin{pmatrix} 4 & 1 \\ 4 & 4 \end{pmatrix} \mathbf{X}, \qquad \mathbf{X}(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}.$$

A. $\mathbf{X} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{2t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{6t}$

B. $\mathbf{X} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{3t}$

C. $\mathbf{X} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{4t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{4t}$

D. $\mathbf{X} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{4t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{4t}$

E. $\mathbf{X} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{2t} - 4 \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{6t}.$

20. A particular solution to the system of equations

$$\mathbf{X}' = \begin{pmatrix} 2 & 1 \\ 0 & -2 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 1 \\ -6 \end{pmatrix} e^t$$

is

A. $\mathbf{X_p} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^t$

B. $\mathbf{X_p} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^t$

C. $\mathbf{X_p} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t}$

D. $\mathbf{X_p} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} e^{-2t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t}$

E. $\mathbf{X_p} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^t - 4 \begin{pmatrix} 4 \\ -1 \end{pmatrix} e^{-2t} - 4 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t}$

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	t^n	$\dfrac{n!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
11.	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \ c > 0$
16.	$\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^nf(t)$	$F^{(n)}(s)$