Mathematics 271

Test Number one

(20 pts) 1. If $\vec{A} = i + 2j - 2k$ and $\vec{B} = 2i + j - k$ find

- a) $\vec{A} \circ \vec{B}$
- b) $\vec{A} \times \vec{B}$
- c) The area of the triangle generated by \vec{A} and \vec{B} .
- d) The length of the projection of \vec{B} onto \vec{A} .
- e) Vectors \vec{B}_1 and \vec{B}_2 such that $\vec{B} = \vec{B}_1 + \vec{B}_2$, \vec{B}_1 is parallel to \vec{A}_1 and \vec{B}_2 is perpendicular to \vec{A} .

2. Which of the following sequences converge? Why? What is the limit? (20 pts)

- a) $a_n = \frac{n^2 2n + 1}{n 1}$
- b) $a_n = (2 \frac{1}{n})(3 + \frac{2}{n})$
- c) $a_n = n^{\frac{2}{n}}$
- d) $a_n = \frac{(\log n)^2}{n}$

3. Which of the following series converge? Why? (20 pts)

- a) $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$ b) $\sum_{n=1}^{\infty} \frac{1+\cos n}{n^2}$ c) $\sum_{n=1}^{\infty} \sin(\frac{1}{n})$

- d) $\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$ e) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n=1\sqrt{n}}$ f) $\sum_{n=1}^{\infty} (-1)^{n+1} (\frac{n+3}{n+5})$

a) For what value of x does the series $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n2^n}$ converge? Where does it (20 pts)converge absolutely.

b) Find Taylor Series in powers of $(x-\frac{\pi}{2})$ for $f(x)=\cos x$, $f(x)=\sin x$, and f(x)=x.

a) Carefully state the Limit Comparison Test for the two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$. (15 pts)

b) Prove that the McLaurin Series for e^x converges for all x.

c) If $f(x) = \sum_{k=0}^{\infty} c_k x^k$ for $|x| < \rho$ prove $c_2 = \frac{f^n(0)}{2!}$.