MA 271 **EXAM 2** **FALL 2002** **Name** ______________________

1. Let $f(x,y) = \left(\frac{x^3}{2} + y\right)^{\frac{2}{3}}$. Find $\frac{\partial f}{\partial x}(4,-5)$.

2. Find $\frac{\partial w}{\partial r}$ when $r = 1$, $s = -1$ if $w = e^{x+y+z}$, $x = r - s$, $y = \cos(r+s)$, $z = \sin(r+s)$.

3. Find the equation of the tangent plane to the surface $y + z - \frac{x^2}{2} = 1$ at $(2,1,2)$.

4. Let $f(x,y,z) = x^2y + y^2z + z^2x$, and $P_0 = (1,1,-1)$. Find the directional derivative of f at P_0 in the direction that the function increases most rapidly.

5. Classify the critical points of $f(x,y) = 4xy - x^4 - y^4$.

6. Find the extreme value of $f(x,y,z) = x + 2y + 2z$ subject to the constraint $x^2 + y^2 + z^2 = 9$.

7. Evaluate $\int_0^1 \int_{\tan^{-1} y}^{\frac{\pi}{4}} \sec^4 x \, dx \, dy$.

8. Let D be the solid region bounded below by the paraboloid $z = x^2 + y^2$ and above by $z = 4$. Evaluate the total mass of D if the density function is $\delta(x,y,z) = \sqrt{x^2 + y^2}$.

9. Evaluate the integral $\iiint_D z \, dv$, where D is the solid region bounded above by the sphere $\rho = 1$ and below by the cone $\phi = \frac{\pi}{6}$.

10. Let D be the image of the rectangle $\{(u,v) \mid -1 \le u \le 2,\ 0 \le v \le 2\}$ under the transformation $x = 2u + 3v$, $y = u - v$. Evaluate $\iint_D \frac{x - 2y}{5} \, dA$.

11. The mass density at a given point of a thin wire C is $\delta(x,y,z) = x$. If C is parametrized by $\mathbf{r}(t) = \langle e^t, 2t, 2e^{-t} \rangle$, $0 \le t \le 1$, find the mass of the wire.

12. Let $\mathbb{F}$ be a conservative vector field given by $\mathbb{F} = \langle 2xy, x^2 + \cos(y + z^2), 3z^2 + 2z\cos(y + z^2) \rangle$. Find f such that $\nabla f = \mathbb{F}$.

13. Let R be the region bounded by $y = x^2$ and $x = y^2$, and C the boundary of R. Compute $\oint_C \mathbb{F} \cdot d\mathbf{r}$ for $\mathbb{F} = \langle 2y + e^x, 3x + \sin y \rangle$.