MA271, Fall 2000
Midterm 3

Instructor: Walther

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- This booklet has SIX QUESTIONS, each worth 10 points. You have 50 minutes to do this test. **Plan your time well. Read the questions carefully.**
- This test is closed books and closed notes.
- Any calculator is allowed. But it cannot be used for justifying answers.
- In order to get full credit, your answers need to be **correct** and **simplified**. You need to explain in a **comprehensible** way how you obtained them.
- Both sides of the paper may be used. **Indicate** which problem you work on whenever you use a backside.

Question 1.
Given is the surface

$$\rho = \cos(\phi)$$

in spherical coordinates.

A) Express the surface in x, y, z-coordinates.

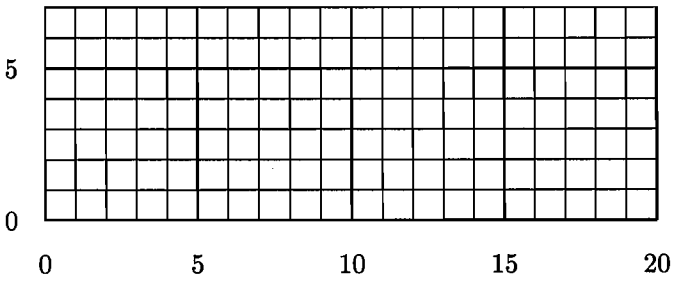
B) Describe the surface in words (as in "it is a hyperbolic paraboloid with axis of rotation equal to the z-axis and inflection point in $(2, 3, 4)$").

C) Compute

$$\int\int_R \int z \, dx \, dy \, dz$$

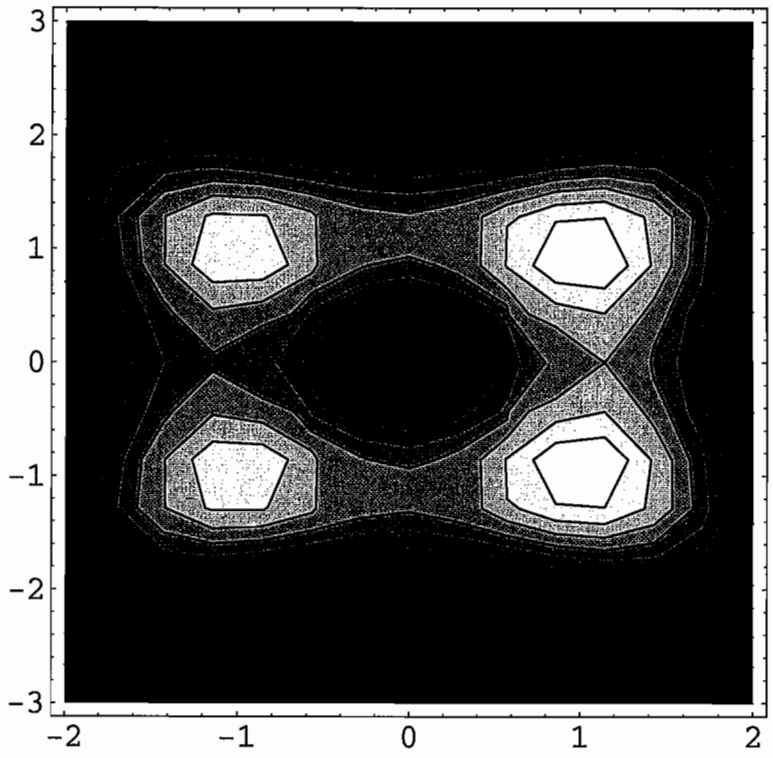
where R is the region enclosed by the surface $\rho = \cos(\phi)$.

Question 2.
Find the center of mass of the shaded area pictured below (assuming constant density equal to 42).



Question 3.

In the picture below we see a topographical map for Wonderland. (That means, drawn are the level curves for the function $(x, y) \mapsto h(x, y)$ where h is the height of the point in Wonderland with coordinates (x, y).) During her travels, Alice comes along the points A, B, C, D and E. Since the sun is just rising, the mountains in Wonderland are lit by the early sun, while the lower parts and valleys are still in the dark.



A) For each of those 5 points, sketch the gradient vector of $h(x, y)$.

B) For each of those 5 points determine whether it is a local max, local min, saddle, or none of the above.

Question 4.

Given is the solid described by $\sqrt{x^2+y^2} \leq z \leq \sqrt{2-x^2-y^2}$.

A) Draw it.

B) Set up the volume integral for the solid.

C) Compute the integral.

Question 5. Find the polar moment of inertia of the circle $x^2 + y^2 = 17$.

Question 6.

A) Compute the flux of the vector field $\vec{v}(x,y) = (x,y)$ across the circle around the origin of radius 3.

B) Compute the flow of the vector field $\vec{v}(x,y) = (x,y)$ along the circle around the origin of radius 3.

C) Comment on the relationship between the function $f(x,y) = x^2 + y^2$ and the integral from part B).