

(10 pts) 1) Find $\frac{\partial w}{\partial x}$ at $(x, y, z) = (2, 1, -1)$ if

$$w = \frac{p - q}{r}, \quad p = -x + y + z,$$

$$q = x - y + z, \quad \text{and } r = x + y + z.$$

(10 pts) 2) If $f(x, y, z) = y^2 + z \ln x$ find

a) ∇f at $(1, 1, 1)$,

b) the direction in which f changes most rapidly at $(1, 1, 1)$,

c) the equation of the tangent plane to $y^2 + z \ln x = 1$ at $(1, 1, 1)$.

(10 pts) 3) Find all maxima, minima, and points of inflection of $f(x, y) = x^3 - y^3 - 2xy + 6$.

(10 pts) 4) Evaluate by reversing the order of integration:

$$\int_0^4 \int_{\frac{y}{2}}^2 e^{x^2} dx dy.$$

(10 pts) 5) Find the average value of $f(x, y) = y \cos x$ in the area bounded by $y = 0$, $y = \sin x$, $0 \leq x \leq \pi$.

(10 pts) 6) If $f(x, y) = y \sin x$ find

a) The linear approximation near $(0, 0)$.

b) The quadratic approximation near $(0, 0)$.

c) An estimate of the error made if f is replaced by its quadratic approximation. Assume $|\Delta x| < 10^{-2}$ and $|\Delta y| < 10^{-2}$.

(10 pts) 7) Find the absolute maximum and minimum of $f(x, y) = x^2 + xy + y^2 - 6x + 2$ on $\{(x, y) | 0 \leq x \leq 5, -3 \leq y \leq 0\}$.

(10 pts) 8) Find the largest product of the positive numbers x , y , and z if

$$x + y + z^2 = 16.$$

(20 pts) 9) Set up but do not evaluate integrals for the following

a) The area inside $t = 2(1 + \sin \theta)$ and outside $r = 1$,

b) I_z (the moment of inertia with respect to the z -axis) of the tetrahedron with corners $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$ if the density $\delta = xy$.

c) The volume between the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 1$ inside $x^2 + y^2 + z^2 = 9$.

d) The volume inside $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 + z^2 = 9$.

e) The area for the smaller part of the region cut from $x^2 + 4y^2 = 12$ by $x = 4y^2$.