MA 271 FINAL EXAM FALL 2002 Name ____________________

Determine whether the given series converge absolutely, converge conditionally, or diverge. Give reasons for your answers.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \tan^{-1} n}{\sqrt[n]{n}}$

2. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \sin(n)}{1+n^2}$

3. $\displaystyle\sum_{n=1}^{\infty} (-1)^n(\sqrt{n+1}-\sqrt{n})$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{3^n}$

5. Find the interval of convergence of
$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n(n+1)}$$

6. Find the Taylor polynomial of xe^{x^2} of order 7 at $x=0$.

7. The approximation $\sin x \sim x - \dfrac{x^3}{3!} + \cdots + \dfrac{(-1)^n x^{2n+1}}{(2n+1)!}$ is used. Determine the smallest n needed to estimate $\sin(0.1)$ with an error of less than 10^{-10}

8. Compute $\displaystyle\lim_{x\to 0} \frac{\ln(1+x^2)}{1-\cos(3x)}$.

9. Find a series solution for $y' - y = x$, $y(0) = 0$.

10. Find the arc length of the curve
$$\mathbf{r}(t) = \langle e^t \sin t, e^t \cos t, e^t \rangle, \quad 0 \le t \le 1.$$

11. Find an equation of the tangent plane to the surface $x - z = y^2$ at $(1,0,1)$.

12. Classify the critical points of the function
$$f(x,y) = x^3 - y^3 - 3xy.$$

13. Find the maximum value of x^3y^2z subjected to the constrain $3x + 2y + z = 12$.

14. Find the area of the image of the rectangle $[0, 2] \times [0, 1]$ under the map $T(u, v) = (u^3 + v, 3v)$.

15. Set up a triple integral for the volume of the solid bounded by the paraboloids $z = 2(x^2 + y^2)$ and $z = 12 - x^2 - y^2$.

16. Let C be the curve given by

    $\mathbf{r}(t) = \langle t^2, t^3, t^4 \rangle, \quad 0 \le t \le 1.$

    Evaluate the line integral

    $$\int_C (xy - z^2)dx + (yz - x^2)dy + (zx - y^2)dz.$$

17. Let C be the circle $x^2 + y^2 = 4$ oriented counterclockwise. Evaluate

    $$\int_C \frac{y\,dx - x\,dy}{x^2 + y^2}.$$

18. Let S be the sphere $x^2 + y^2 + z^2 = 4$ with outward normal and

    $\mathbf{F} = \langle xy^2 + z, yz^2 + x, zx^2 + y \rangle$. Compute $\iint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma$.

19. Let S be the portion of the cone $z^2 = x^2 + y^2$ with $0 \le z \le 2$. Compute

    $$\iint_S z^2 d\sigma$$

20. Let S be the portion of the paraboloid $z = x^2 + y^2$, $z \le 4$ with downward normal and $\mathbf{F} = \langle xz, yz, xy \rangle$. Compute

    $$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, d\sigma.$$