MA 16100 Exam I, Spring 2014, February 13

Name ____

10–digit PUID number ________________________________

Recitation Instructor ________________________________

Recitation Section Number and Time ________________________

Instructions: MARK TEST NUMBER O1 ON YOUR SCANTRON

1. Do not open this booklet until you are instructed to.
2. Fill in all the information requested above and on the scantron sheet. On the scantron sheet fill in the little circles for your name, section number and PUID.
3. This booklet contains 14 problems, equally weighted.
4. For each problem mark your answer on the scantron sheet and also **circle it in this booklet**.
5. Work only on the pages of this booklet.
6. Books, notes, calculators or any electronic device are not allowed during this test and they should not even be in sight in the exam room. You may not look at anybody else's test, and you may not communicate with anybody else, except, if you have a question, with your instructor.
7. You are not allowed to leave during the first 20 and the last 10 minutes of the exam.
8. When time is called at the end of the exam, put down your writing instruments and remain seated. The TAs will collect the scantrons and the booklets.

1. Suppose k is a given number. The equation of the line through the points $(1, 2)$ and $(2, k)$ is

$y-2=\frac{k-2}{2-1}(x-1)$

$y-2=(k-2)(x-1)=(k-2)x-(k-2)$

$y=(k-2)x - k+2+2 = \underline{(k-2)x+4-k}$

A. $y = (k-2)x + 4 - k$

B. $y = (k-2)x - 4 + k$

C. $y = (k+2)x + 4 - k$

D. $y = (k-2)x + 4 + k$

E. $y = (k+2)x - 4 + k$

2. $\displaystyle\lim_{x\to\infty} \frac{x^2+3}{\sqrt{2x+1}} =$

$\frac{x^2+3}{\sqrt{2x+1}} = \frac{x^2+3}{\sqrt{x}\sqrt{2+1/x}} = \frac{\overset{\to\infty}{x^{3/2}} + \overset{\to 0}{\frac{3}{\sqrt{x}}}}{\underset{\searrow\sqrt{2}}{\sqrt{2+\frac{1}{x}}}}$ as $x \to \infty$.

A. $1/\sqrt{2}$

B. $3/\sqrt{2}$

C. 1

D. $-\infty$

E. ∞

3. The expression $\lim_{h\to 0} \dfrac{\sqrt{4+h}-2}{h}$ represents the derivative of which function f at which number a?

2

I. $f(x) = \sqrt{x}$ at $a = 4$; $\lim_{0} \dfrac{\sqrt{4+h}-\sqrt{4}}{h}$ ✓

II. $f(x) = \sqrt{x} - 2$ at $a = 4$; $\lim_{0} \dfrac{(\sqrt{4+h}-2)-(\sqrt{4}-2)}{h}$ ✓

III. $f(x) = \sqrt{2+x}$ at $a = 2$

$\lim_{0} \dfrac{\sqrt{2+2+h}-\sqrt{2+2}}{h}$ ✓

A. only I

B. only II

C. only III

D. only I and II

E. I, II and III

4. If $g(x) = \dfrac{1+3x}{5-2x}$, $g^{-1}(1) =$

$\dfrac{1+3x}{5-2x} = 1$

$1+3x = 5-2x$

$5x = 4$

$x = 4/5$

A. 2/5

B. 3/5

C. 4/5

D. 6/5

E. 9/5

5. The graph of $F(x) = 4 - 2^{x/3}$ is obtained from the graph of $G(x) = 2^x$ by the following steps:

A. Compress horizontally by a factor of 3, then reflect about the x axis, then shift up by 4 units.

B. Stretch horizontally by a factor of 3, then reflect about the x axis, then shift up by 4 units.

C. Compress horizontally by a factor of 3, then reflect about the y axis, then shift up by 4 units.

D. Stretch horizontally by a factor of 3, then reflect about the y axis, then shift down by 4 units.

E. Stretch horizontally by a factor of 3, then reflect about the y axis, then shift to the right by 4 units.

$2^{x/3}$: Stretch horiz by factor 3

$-2^{x/3}$, Then reflect about x axis

$-2^{x/3}+4$: " shift up by 4

6. $\lim_{t \to 0^+} \frac{25}{t^2} - \frac{1}{10t^3} =$

A. 75

B. 10

C. 3

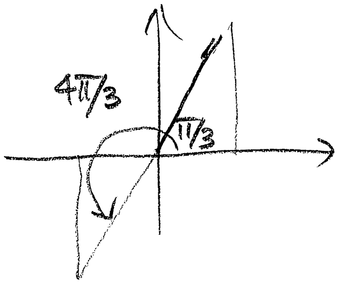
D. ∞

E. $-\infty$

$$\frac{25}{t^2} - \frac{1}{10t^3} = \left(\frac{1}{t^3}\right)\left(\overbrace{25t - \frac{1}{10}}^{\to -1/10}\right)$$

$\frac{1}{t^3} \downarrow \infty$ as $t \to 0^+$

7. Evaluate $\cos \pi/3 - \sin 4\pi/3$.



$\cos\frac{\pi}{3} = \cos 60^\circ = \frac{1}{2}$

$\sin\frac{4\pi}{3} = -\sin\frac{\pi}{3} = -\sin 60^\circ = \frac{-\sqrt{3}}{2}$

A. $\dfrac{1-\sqrt{3}}{2}$

B. $\dfrac{1-\sqrt{2}}{2}$

C. 0

D. $\dfrac{1+\sqrt{2}}{2}$

E. $\dfrac{1+\sqrt{3}}{2}$

8. Which is true?

$\ln\frac{1}{e} = -1$ (for $e^{-1} = \frac{1}{e}$)

$\log_4 16 = 2$ (for $4^2 = 16$)

$\log_3 \sqrt[4]{3} = \frac{1}{4}$ (for $3^{1/4} = \sqrt[4]{3}$)

A. $\ln\dfrac{1}{e} < \log_4 16 < \log_3 \sqrt[4]{3}$

B. $\log_3 \sqrt[4]{3} < \ln\dfrac{1}{e} < \log_4 16$

C. $\ln\dfrac{1}{e} < \log_3 \sqrt[4]{3} < \log_4 16$

D. $\log_4 16 < \log_3 \sqrt[4]{3} < \ln\dfrac{1}{e}$

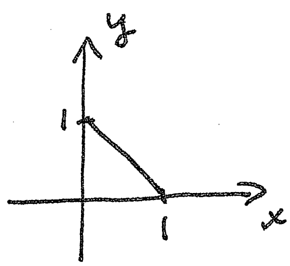
E. $\log_3 \sqrt[4]{3} < \log_4 16 < \ln\dfrac{1}{e}$

9. The domain of the function $\varphi(x) = \ln(1 - |x|)$ is

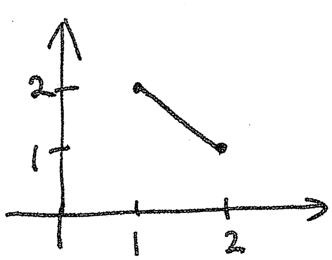
A. $(-1, 1)$
B. $[-1, 1]$
C. $(0, \infty)$
D. $(0, 1)$
E. $[0, 1]$

Need $1 - |x| > 0$

$1 > |x|$

10. If the graph $y = f(x)$ is 

, which function has graph



?

A. $f(x+1)+1$
B. $f(x-1)+1$
C. $f(x+1)-1$
D. $f(x-1)-1$
E. $f(1-x)+1$

Shift right by 1 : $f(x-1)$

" up " : $f(x-1)+1$

11. $\lim_{y \to 3} \dfrac{y^2 - 4y + 3}{y^2 - 7y + 10} =$

$$\frac{y^2-4y+3}{y^2-7y+10}$$

$\to 9-12+3=0$

$\to 9-27+10=-8$

A. 0

B. 1

C. 2

D. 3

E. The limit does not exist

12. A camel is galloping along a straight road, its position at time t is given by the function $p(t) = 6 + 8t - e^{2-t}$. What is its average velocity over the time period $[0, 2]$?

$$\frac{p(2)-p(0)}{2-0} = \frac{(6+8\cdot 2-1)-(6+0-e^2)}{2}$$

$$= \frac{15+e^2}{2}$$

A. $6 - e$

B. $14 - e^2$

C. $\dfrac{15 + e^2}{2}$

D. $\dfrac{14 + e^2}{4}$

E. $\dfrac{8 + e}{2}$

13. $\lim_{s\to 1} \frac{s^2-6s+5}{s^2-1} =$ $\to 0$ $\to 0$

A. -5

B. -2

C. 1

D. ∞

E. 3

$$\frac{s^2-6s+5}{s^2-1} = \frac{\cancel{(s-1)}(s-5)}{\cancel{(s-1)}(s+1)} \to \frac{-4}{2}$$

as $s \to 1$

14. For which choice of the number b is the function

$$H(z) = \begin{cases} 2z+b, & \text{if } z < -1 \\ z^2, & \text{if } z \geq -1 \end{cases}$$

continuous on $(-\infty, \infty)$?

A. 3

B. 2

C. 1

D. 0

E. There is no such b.

Only possible discontinuity at $z=-1$.

There

$$\lim_{z\to -1^-} H(z) = \lim_{z\to -1^-} 2z+b = -2+b,$$

$$\lim_{z\to -1^+} H(z) = \lim_{z\to -1^+} z^2 = 1 = H(1).$$

Continuous at $z=-1$ if $1=-2+b$.