MA 161 EXAM II **Spring 2004**

Name ______________________________

nine–digit Student ID number ______________________________

Division and Section Numbers ______________________________

Recitation Instructor ______________________________

Instructions:

1. Fill in all the information requested above and on the scantron sheet.
2. This booklet contains 20 problems, each worth 5 points.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
4. Work only on the pages of this booklet.
5. Books, notes, calculators are not to be used on this test.
6. At the end turn in your exam and scantron sheet to your recitation instructor.

cabd badc edbe ccea

deab

1. The tangent line to $y = 2x + 8\sqrt{x}$ at $x = 4$ is

   a. $y = 3(x-4) + 20$

   b. $y = 4(x-4) + 20$

   c. $y = 4(x-4) + 24$

   d. $y = 5(x-4) + 24$

   e. none of these

$y' = 2 + \frac{4}{x^{1/2}}$, $y'(4) = 4$, $y(4) = 24$

2. If $f(x) = e^{2x}\tan^{-1}(4x)$ then $f'(x)$ equals

   a. $2e^{2x}\tan^{-1}(4x) + \frac{e^{2x}4}{1+(4x)^2}$

   b. $2e^{2x}\tan^{-1}(4x) + \frac{e^{2x}}{1+(4x)^2}$

   c. $e^{2x}\tan^{-1}(4x) + \frac{e^{2x}}{1+(4x)^2}$

   d. $2e^{2x}\tan^{-1}(4x) + \frac{e^{2x}4}{1+x^2}$

   e. $2e^{2x} + \frac{4}{1+(4x)^2}$

3. Let

$$\begin{array}{lll} f(1) = 1, & f(2) = 2, & f(3) = 1 \\ f'(1) = 0, & f'(2) = 6, & f'(3) = 0 \\ g(4) = 2, & g(5) = 4, & g(6) = 8 \\ g'(4) = 7, & g'(5) = 2, & g'(6) = 1. \end{array}$$

Then $\frac{d}{dx} f \circ g(4)$ equals

   a. 0

   b. 42

   c. 12

   d. 6

   e. 48

$f'(g(4)) \cdot g'(4) = f'(2) \cdot g'(4)$

4. If $y(x)$ solves $4\cos x \sin y = \sqrt{2}$ and $y(\frac{\pi}{3}) = \frac{\pi}{4}$ find $y'(\frac{\pi}{3})$.

a. $-2\sqrt{3}$

b. $-\sqrt{3}/2$

c. $-\sqrt{3}$

d. $\sqrt{3}$

e. $\sqrt{3}/2$

$-\sin x \sin y + \cos x \cos y \; y' = 0$

$y' = \frac{\sin x}{\cos x} \cdot \frac{\sin y}{\cos y}$, $x = \frac{\pi}{3}$, $y = \frac{\pi}{4}$

5. If the average cost of producing x items is $A(x) = \dfrac{\ln(1+x^2)}{x^2}$ and $C(x)$ is the total cost then $C'(x)$ is

a. $\dfrac{\ln(1+x^2)}{x^2}$

b. $\dfrac{2}{1+x^2} - \dfrac{\ln(1+x^2)}{x^2}$

c. $\dfrac{2x}{1+x^2} - \dfrac{\ln(1+x^2)}{x^2}$

d. $\dfrac{2}{x^2(1+x^2)} - \dfrac{3\ln(1+x^2)}{x^4}$

e. $-\dfrac{\ln(1+x^2)}{x^2}$

$C(x) = x \cdot A(x) = \dfrac{\ln(1+x^2)}{x}$

$C'(x) =$

6. If $g(t) = \sin^{-1}\left(\dfrac{1}{t}\right)$ then $g'(t)$ equals

a. $\dfrac{-1}{t\sqrt{t^2-1}}$

b. $\dfrac{-1}{\sqrt{t^2-1}}$

c. $\dfrac{t^2}{\sqrt{t^2-1}}$

d. $\dfrac{-t^2}{\sqrt{t^2-1}}$

e. $\dfrac{t}{\sqrt{t^2-1}}$

$g'(t) = \dfrac{1}{\sqrt{1-(\frac{1}{t})^2}} \cdot \left(\dfrac{-1}{t^2}\right)$

7. $\lim_{x\to\infty} (\ln(6x^2+1) - \ln(2x^2+4))$ equals

a. ∞

b. $-\infty$

c. $\ln \frac{3}{2}$

d. $\ln\ 3$

e. $\ln\ \frac{3}{4}$

$\lim_{x\to\infty} \ln\left(\frac{6x^2+1}{2x^2+4}\right)$

8. If $y(x)$ satisfies $y^2 - x^2 = -3$ and $y(1) = 2$ then $y''(1)$ equals

a. 3

b. 3/2

c. 3/8

d. 3/4

e. 3/16

$yy' - x = 0$, $y' = \frac{x}{y}$

$y'' = \frac{y - xy'}{y^2} = \frac{y - \frac{x^2}{y}}{y^2}$

9. For what values of λ does $y(t) = e^{\lambda t}$ satisfy $y'' + y' - 2y = 0$?

a. $\{-1, 2\}$

b. $\{0, 3\}$

c. $\{0, -3\}$

d. $\{1, 2\}$

e. $\{1, -2\}$

$y' = \lambda e^{\lambda t}$, $y'' = \lambda^2 e^{\lambda t}$

$\lambda^2 e^{\lambda t} + \lambda e^{\lambda t} - 2e^{\lambda t} = 0$

$e^{\lambda t}(\lambda^2 + \lambda - 2) = 0$

$(\lambda + 2)(\lambda - 1) = 0$

10. If $y = x\ln(x)$ for $x > 0$, for what x does $y'(x) = 0$?

a. 0

b. 1

c. e

d. e^{-1}

e. none of these

$y' = \ln x + 1$

$\ln x = -1 \approx \quad x = e^{-1}$

11. If $y = \tan^{-1}(x)$ then $y''(1)$ equals

a. $\frac{1}{2}$

b. $-\frac{1}{2}$

c. $-\frac{1}{4}$

d. $\frac{1}{4}$

e. none of these

$y' = \frac{1}{1+x^2}, \quad y'' = \frac{-2x}{(1+x^2)^2}$

12. $\lim_{x\to\infty} \tanh(x)$ equals

a. -1

b. $-\infty$

c. ∞

d. 0

e. 1

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \cdot \frac{(e^{-x})}{(e^{-x})}$

$= \frac{1 - e^{-2x}}{1 + e^{-2x}}$

13. If $y = (x+2)^4(3x+1)^6$ then $(\ln y)'$ equals

a. $\dfrac{4}{x+2} + \dfrac{6}{3x+1}$

b. $4(x+2) + 6(3x+1)$

c. $\dfrac{4}{x+2} + \dfrac{18}{3x+1}$

d. $4\ln(x+2) + 6\ln(3x+1)$

e. $24\ln(x+2)\ln(3x+1)$

$\ln y = 4\ln(x+2) + 6\ln(3x+1)$

14. If $y = x^x$ then $y'(x)$ equals

a. $\dfrac{x^x}{x}$

b. $\ln x + 1$

c. $x^x(\ln x + 1)$

d. $x^x \ln x$

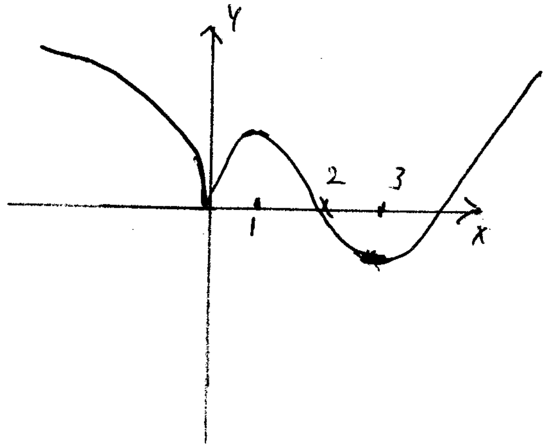
e. $\ln x$

$\ln y = x\ln x$

$\dfrac{y'}{y} = \ln x + 1$

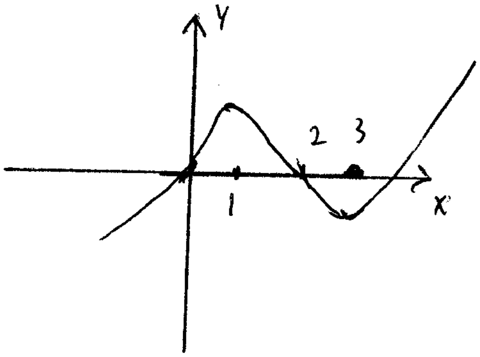
$y' = x^x(\ln x + 1)$

15. If the graph of f is



then the graph of f' is given by

a.



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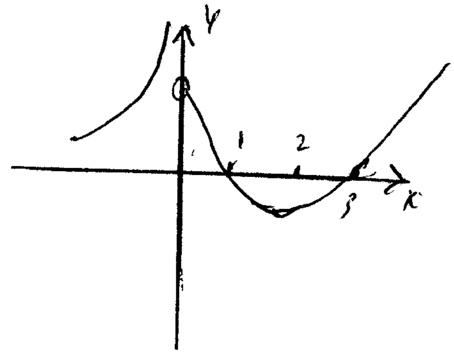
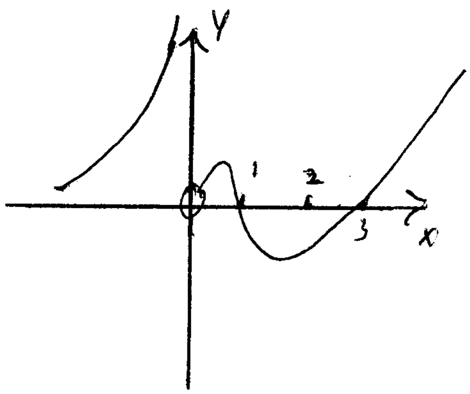
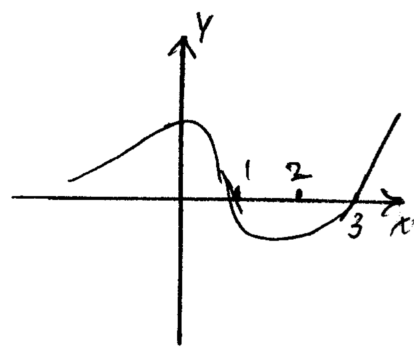


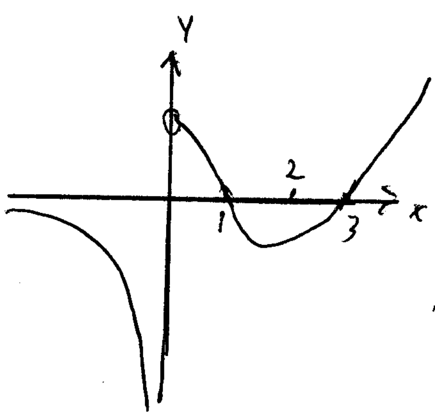
c.



d.



e.



16. Determine the limit $\lim_{x\to\infty} \left(\frac{8x^2+2x+1}{2x^2+4x+1}\right)^{1/2}$.

a. 2

b. 4

c. 8

d. $\sqrt{2}$

e. $\sqrt{8}$

$$\lim_{x\to\infty} \frac{8x^2+2x+1}{2x^2+4x+1} = \frac{8}{2} = 4$$

$$\text{so } (\quad)^{1/2} \to 4^{1/2} = 2$$

17. If an object's position at time t is $s(t) = t^4 - 24t^2$ for $t > 0$. For what time t is the acceleration zero?

a. $\sqrt{24}$

b. $\sqrt{12}$

c. $\sqrt{8}$

d. 2

e. $\sqrt{2}$

$$v = s' = 4t^3 - 48t$$

$$a(t) = s'' = 12t^2 - 48$$

18. Which functions have removable discontinuities at $t = 2$?

$$f(x) = \begin{cases} 2x+1 & x > 2 \\ x+3 & x < 2 \end{cases}, \quad g(x) = \frac{x^2-x-2}{x-2} \quad x \neq 2, \quad h(x) = x^2 \quad x \neq 2.$$

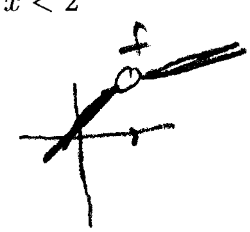
a. just f

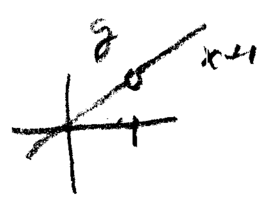
b. just f and g

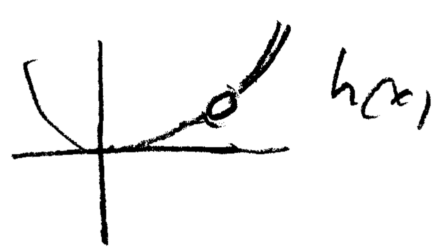
c. just f and h

d. just g and h

e. all of these







19. Which functions have a vertical asymptote at $x = 2$?

$$f(x) = \frac{x^2 + 3x + 2}{x - 2} \quad x \neq 2, \quad g(x) = \frac{x^2 - 3x + 2}{x - 2} \quad x \neq 2, \quad h(x) = \begin{cases} x + 1 & x > 2 \\ x - 2 & x < 2. \end{cases}$$

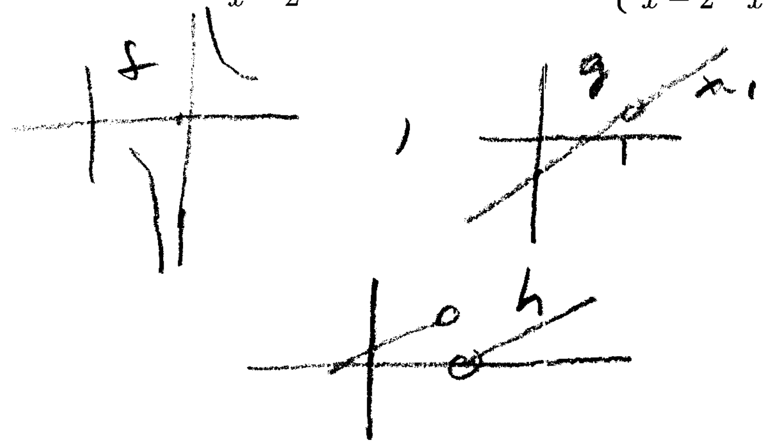
a. just f

b. just f and g

c. just f and h

d. just h and g

e. all of these



20. A rectangle's sides vary with t, $x = x(t)$, $y = y(t)$. Find the rate of change of the rectangle's area with respect to t if at time t

$$x = 2, \; y = 3, \; x' = 4, \text{ and } y' = 5.$$

a. 6

b. 22

c. 23

d. 26

e. 120

$A(t) = x(t)\, y(t)$

$A' = x'y + xy'$