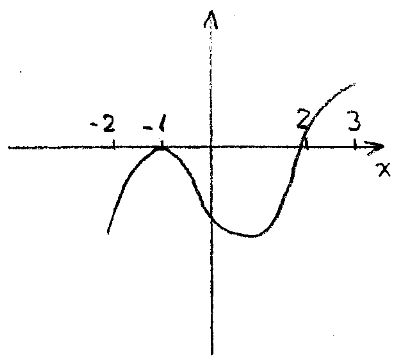
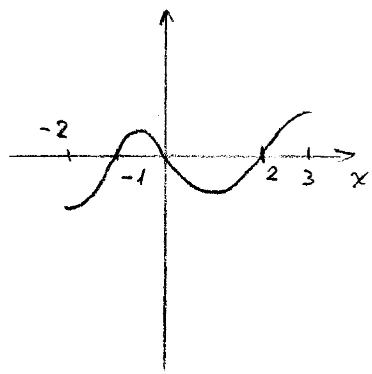
KEY: EBDEE DAEAB BA

MA 161 & 161E — EXAM 3 — November 2003

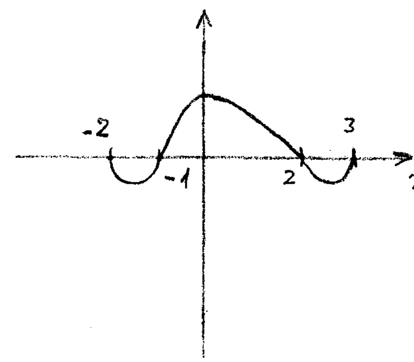
1. Given that $\overbrace{f'(x) > 0}^{\text{increasing}}$ when $-1 < x < 0$ and $\boxed{2 < x} < 3$, and $\overbrace{f'(x) < 0}^{\text{decreasing}}$ when $-2 < x < -1$ and $0 < \boxed{x < 2}$, which could be the graph of f? local minimum at $x=2$



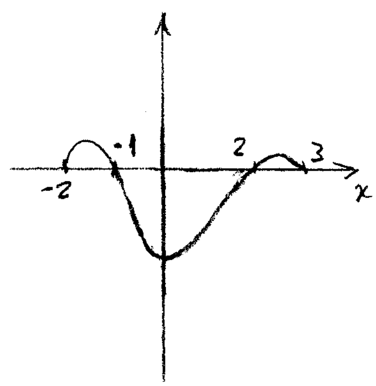
A.



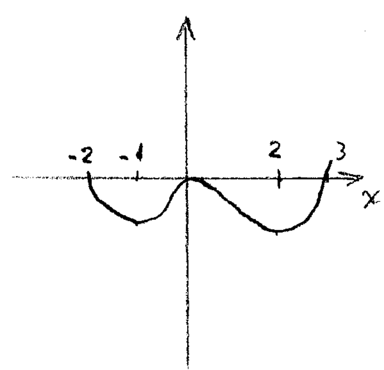
B.



C.



D.



E.

2. The derivative of a function g is $g'(x) = \sin x - \sin 2x$, so that $x = 0$ and $x = \pi/3$ are critical numbers of g. Then, g has

A. a local minimum at 0 and a local maximum at $\pi/3$

B. a local maximum at 0 and a local minimum at $\pi/3$

C. a local maximum at 0 and an inflection point at $\pi/3$

D. a local maximum at $\pi/3$

E. inflection points at $0, \pi/3$

$g''(x) = \cos x - 2\cos 2x$

$g''(0) = 1 - 2 < 0$

$g''(\frac{\pi}{3}) = \frac{1}{2} - 2(-\frac{1}{2}) > 0$

3. $\lim_{x\to\infty} \frac{\ln x}{e^{2x}} = \lim_{x\to\infty} \frac{1}{2x\, e^{2x}} = 0$

A. ∞

B. e

C. 1

D. 0

E. -1

4. $\lim_{x\to 0} \frac{1-\cos 2x}{x^2} = \lim_{x\to 0} \frac{2\sin 2x}{2x} = \lim_{x\to 0} \frac{2\cos 2x}{1} = 2$

A. 0

B. ∞

C. $\frac{\pi}{2}$

D. 1

E. 2

5. $\lim_{x\to 0^+} (1+\sin x)^{1/x} =$ $e^{\lim_{x\to 0^+} \frac{1}{x} \ln(1+\sin x)}$

$= e^{\lim_{x\to 0} \frac{\cos x}{1+\sin x}} = e^{\frac{1}{1+0}} = e$

A. 0

B. ∞

C. $\ln 2$

D. 2

E. e

6. The minimum value of $f(x) = 3x + \dfrac{12}{x^2}$ for $x > 0$ is

$f'(x) = 3 - \frac{24}{x^3} = 0 \iff x^3 = 8 \iff x = 2$

$f(2) = 9$

$\lim_{x\to 0^+} f(x) = \infty$

$\lim_{x\to \infty} f(x) = \infty$

A. 6

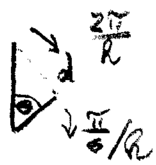
B. 8

C. $\dfrac{26}{3}$

D. 9

E. 10

7. The minute hand on a watch is 2 in long and the hour hand is 1 in long. At two o'clock the distance between the tips of the hands is $\sqrt{3}$ in. How fast is the distance between the tips of the hands decreasing at that moment?



$\theta' = \frac{d\theta}{dt} = -\frac{11\pi}{6}$

$d^2 = 2^2 + 1^2 - 2(1)(2)\cos\theta$

$2dd' = 4\sin\theta\,\theta'$

$d' = \dfrac{4\frac{\sqrt{3}}{2}\frac{11\pi}{6}}{2\sqrt{3}} = \dfrac{11\pi}{6}$

A. $\frac{11\pi}{6}$ in/hour

B. $\frac{11\pi\sqrt{3}}{6}$ in/hour

C. $\frac{11\pi}{12}$ in/hour

D. $\frac{11\pi\sqrt{3}}{12}$ in/hour

E. $\frac{11\pi}{6\sqrt{3}}$ in/hour

8. The linear approximation of $f(x) = x^{20}$ at $a = 20$ is used to find an approximate value for 19^{20}. The approximate value found is

$f'(x) = 20x^{19}$

$L(x) = f(20) + f'(20)(x-20)$

$x = 19$

$19^{20} \approx 20^{20} - 20^{20} = 0$

A. 19^{19}

B. 19^{20}

C. -19^{19}

D. 20^{19}

E. 0

9. Suppose that f is continuous on $[2,5]$ and $2 \leq f'(x) \leq 5$ for all x in $(2,5)$. Then, the mean value theorem implies that $f(5) - f(2)$ lies in the interval

$3 \times 2 \leq f(5) - f(2) = (5-2) f'(c) \leq 3 \times 5$

A. $[6, 15]$

B. $[3, 12]$

C. $[2, 5]$

D. $[0, 5]$

E. $[-5, 5]$

10. The critical numbers of $R(t) = t^{1/3} - t^{-2/3}$ are

$t \neq 0$

$R'(t) = \frac{1}{3} t^{-2/3} + \frac{2}{3} t^{-5/3}$

$= \frac{t^{-5/3}}{3} (t+2) = 0$

$\Leftrightarrow t = -2$

A. 0 and 2

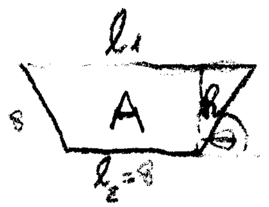
B. -2 only

C. 0 and $\pm\sqrt{3}$

D. -2 and -1

E. 2 and $\pm\sqrt{3}$

11. A rain gutter is to be constructed from a metal sheet of width 24 cm by bending up one-third of the sheet on each side through an angle θ. In order to choose θ so that the gutter will carry the maximum amount of water, the function to be maximized is



$$A = \frac{h}{2}(l_1 + l_2)$$

$$= \frac{8\sin\theta}{2}(8 + 16\cos\theta + 8)$$

$$= 64(\sin\theta\cos\theta + \sin\theta)$$

A. $64(\cos^2\theta + \cos\theta)$

B. $64(\sin\theta\cos\theta + \sin\theta)$

C. $32\sin^2\theta + 16\cos^2\theta$

D. $32(\sin^2\theta + \sin\theta\cos\theta)$

E. $32\cos^2\theta + 16\sin\theta\cos\theta$

12. The total number of local maxima, local minima, and inflection points in the graph of $f(x) = \dfrac{1}{1-x^2}$ is

A. 1

B. 2

C. 3

D. 4

E. 5

$$f'(x) = +2x(1-x^2)^{-2} = \frac{2x}{(1-x^2)^2} \Rightarrow x = 0 \text{ loc. min.}$$

$$f''(x) = \frac{2(1-x^2)^2 - 2x\,2(1-x^2)(-2x)}{(1-x^2)^4}$$

$$= \frac{2(1-x^2)[1-x^2+4x^2]}{(1-x^2)^4}$$

$$= \frac{6x^2+2}{(1-x^2)^3}$$

$f''(x) > 0$ for $x \in (-1,1)$

$f''(x) < 0$ for $x \in (-\infty,-1) \cup (1,\infty)$

NO INFLECTION POINTS ($x = \pm 1$ are not in domain of f)