MA 16600
EXAM 3 INSTRUCTIONS
VERSION 01
April 17, 2014

Your name ____________________ Your TA's name ____________________

Student ID # ________________ Section # and recitation time __________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your question booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the spaces provided for each of the questions 1–12. Do all your work on the question sheets. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

8. There are 12 questions, each worth 8 points. The maximum possible score is

$8 \times 12 + 4$ (for taking the exam) $= 100$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheets and the exam booklets.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____________________________________

Questions

1. Compute

$$\sum_{n=1}^{\infty} \frac{2^{n-1} - (-3)^{n+1}}{4^n}.$$

A. $\frac{11}{5}$

B. $\frac{13}{3}$

C. $-\frac{11}{14}$ (correct)

D. $-\frac{7}{2}$

E. $\frac{3}{8}$

2. The series

$$\sum_{n=1}^{\infty} \frac{1}{(n^{2p}+1)^{1/6}}$$

is convergent if and only if

A. $p > \frac{1}{6}$

B. $p \geq 3$

C. $p \geq \frac{1}{6}$

D. $p > 3$ (correct)

E. $p \neq \frac{1}{2}$.

3. Test the following series for convergence or divergence.

(a) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

(b) $\sum_{n=1}^{\infty} (-1)^n \arctan n$

(c) $\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}}$.

A. (a) convergent (b) convergent (c) convergent

B. (a) convergent (b) divergent (c) convergent

C. (a) divergent (b) convergent (c) convergent

D. (a) divergent (b) convergent (c) divergent

E. (a) divergent (b) divergent (c) divergent (correct)

4. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

(a) $\sum_{n=1}^{\infty}(-1)^{n-1}\ \frac{2n}{3n+5}$

(b) $\sum_{n=1}^{\infty}(-1)^{n-1}\ \frac{1}{n^3}$

(c) $\sum_{n=2}^{\infty}(-1)^{n-1}\ \frac{\ln n}{n}$.

A. (a) absolutely convergent (b) conditionally convergent (c) divergent

B. (a) conditionally convergent (b) absolutely convergent (c) divergent

C. (a) divergent (b) absolutely convergent (c) conditionally convergent (correct)

D. (a) conditionally convergent (b) conditionally convergent (c) divergent

E. (a) divergent (b) divergent (c) absolutely convergent

5. Test the following series for convergence or divergence.

(a) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(b) $\sum_{n=1}^{\infty} \left(\frac{3n+2}{2n+3}\right)^n$

(c) $\sum_{n=1}^{\infty} \frac{n}{5^n}$.

A. (a) convergent (b) convergent (c) convergent
B. (a) convergent (b) divergent (c) convergent (correct)
C. (a) divergent (b) convergent (c) convergent
D. (a) divergent (b) convergent (c) divergent
E. (a) divergent (b) divergent (c) divergent

6. Which of the following statements are **always true**?

I. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

II. If $\lim_{n\to\infty} n^3|a_n| = 0$, then $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges.

III. $\sum_{n=1}^{\infty} \frac{e^n + c}{e^{2n}}$ converges for any positive value c.

A. I only.

B. II only.

C. III only.

D. I and II only.

E. II and III only (correct)

7. Given the following series

$$\sum_{n=1}^{\infty} \frac{3}{2^n + n - 1},$$

Mark, Nancy, and David provide the following ingredients of the arguments for convergence or divergence of the series:

(a) the name of the test to use,

(b) the conclusion for convergence or divergence.

Mark: (a) $b_n = \frac{3}{2^n}$, Comparison Test ($0 \leq a_n \leq b_n$) (b) convergent

Nancy: (a) $b_n = \frac{1}{n}$, Limit Comparison Test ($\lim_{n\to\infty} \frac{a_n}{b_n} = 3$) (c) divergent

David: (a) Ratio Test ($\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \frac{1}{2}$) (b) convergent

Choose the name(s) of the person(s) with correct arguments.

A. Mark only.

B. Nancy only.

C. David only.

D. Mark and David only. (correct)

E. Nancy and David only.

8. Consider the Maclaurin series for e^x

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots .$$

By plugging in $x = -1$, one obtains the alternating series

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \cdots$$

If we compute the sum of the **fewest** terms necessary to guarantee that the error is less than 0.05, **using Estimation Theorem for Alternating Series**, then what is the estimate for e^{-1} ?

A. $\frac{11}{8}$

B. $\frac{3}{8}$

C. $\frac{3}{7}$

D. $\frac{2}{5}$

E. $\frac{1}{3}$ (correct)

9. Suppose the power series

$$\sum_{n=0}^{\infty} c_n(x-3)^n$$

converges when $x = 1$ but diverges when $x = 7$.

From the above information, which of the following statements can we conclude to be true ?

I. The radius of convergence is $R \geq 2$.

II. The power series converges at $x = 4.5$

III. The power series diverges at $x = 6.5$

A. I and II only. (correct)

B. I and III only.

C. II and III only.

D. All of them

E. None of them

10. What is the coefficient of x^6 in the power series expansion of $\frac{2}{(1+2x^2)}$?

A. 8

B. -8

C. 32

D. -16 (correct)

E. -64

11. Determine the interval of convergence for the following power series

$$\sum_{n=0}^{\infty} \frac{(-5)^n}{\sqrt{n+2}}(x-3)^n.$$

A. $(-5+3, 5+3)$

B. $(-\frac{1}{5}, \frac{1}{5}]$

C. $(-\frac{1}{5}+3, \frac{1}{5}+3)$

D. $(-\frac{1}{5}+3, \frac{1}{5}+3]$ (correct)

E. $[-\frac{1}{5}+3, \frac{1}{5}+3)$

12. The power series representation (centered at $a = 0$) for $g(x) = \frac{x}{4-x^2}$ is given by

$$g(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{4^{n+1}}$$

with the interval of convergence $(-2, 2)$.

Find

(a) the power series representation (centered at $a = 0$), and

(b) the interval of convergence

for the function

$$f(x) = \ln|4 - x^2|.$$

A. (a) $-\sum_{n=0}^{\infty} \frac{x^{2n+2}}{(2n+2)4^{n+1}}$ (b) $(-2, 2)$

B. (a) $-2\sum_{n=0}^{\infty} \frac{x^{2n+2}}{(2n+2)4^{n+1}}$ (b) $(-2, 2)$

C. (a) $\ln 4 - 2\sum_{n=0}^{\infty} \frac{x^{2n+2}}{(2n+2)4^{n+1}}$ (b) $(-2, 2)$ (correct)

D. (a) $\ln 4 - 2\sum_{n=0}^{\infty} \frac{x^{2n+2}}{(2n+2)4^{n+1}}$ (b) $[-2, 2)$

E. (a) $\ln 4 - \frac{1}{2}\sum_{n=0}^{\infty} \frac{x^{2n+2}}{(2n+2)4^{n+1}}$ (b) $(-2, 2)$