Student: ____________________ **Date:** ____________________	**Instructor:** Tong Ding, Purdue Math, Jennifer Hobbs, Stephanie Foster **Course:** MA166-123- Analytic Geom. And Calc. II-Spring 2020	**Assignment:** Exam3

1. Determine whether the following series converges absolutely, converges conditionally, or diverges.

$$\sum_{k=1}^{\infty}(-1)^k a_k = \sum_{k=1}^{\infty}\frac{(-1)^k k^5}{\sqrt{k^{12}+16}}$$

Find $\lim_{k\to\infty} a_k$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $\lim_{k\to\infty} a_k =$ ________
- **B.** The limit does not exist.

Now, let $\sum a_k$ denote $\sum_{k=1}^{\infty}\frac{(-1)^k k^5}{\sqrt{k^{12}+16}}$. What can be concluded from this result using the Divergence Test?

- **A.** The series $\sum a_k$ must diverge.
- **B.** The series $\sum |a_k|$ must converge.
- **C.** The series $\sum |a_k|$ must diverge.
- **D.** The series $\sum a_k$ must converge.
- **E.** The Divergence Test is inconclusive.

Are the terms of the sequence $|a_k|$ decreasing after some point?

- yes
- no

Let $\sum a_k$ denote $\sum_{k=1}^{\infty}\frac{(-1)^k k^5}{\sqrt{k^{12}+16}}$. What can be concluded from these results using the Alternating Series Test?

- **A.** The series k^6 must diverge.
- **B.** The series $\sum a_k$ must diverge.
- **C.** The series k^6 must converge.
- **D.** The series $\sum a_k$ must converge.
- **E.** The Alternating Series Test does not apply to this series.

Does the series $\sum |a_k|$ converge?

- **A.** yes, as can be determined by the Limit Comparison Test
- **B.** no, as can be determined by the Limit Comparison Test
- **C.** no, because of the Divergence Test

Does the series $\sum a_k$ converge absolutely, converge conditionally, or diverge?

- **A.** The series converges conditionally because $\sum |a_k|$ converges but $\sum a_k$ diverges.
- **B.** The series diverges because $\sum |a_k|$ diverges.
- **C.** The series converges conditionally because $\sum a_k$ converges but $\sum |a_k|$ diverges.
- **D.** The series converges absolutely because $\sum |a_k|$ converges.
- **E.** The series diverges because $\lim_{k\to\infty} a_k \neq 0$.

Answers A. $\lim_{k\to\infty} a_k =$ **0**

E. The Divergence Test is inconclusive.

yes

D. The series $\sum a_k$ must converge.

B. no, as can be determined by the Limit Comparison Test

C. The series converges conditionally because $\sum a_k$ converges but $\sum |a_k|$ diverges.

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2. Find the power series representation for g centered at 0 by differentiating or integrating the power series for f (perhaps more than once). Give the interval of convergence for the resulting series.

$$g(x) = \mathbf{ln}\,(1 - 9x) \text{ using } f(x) = \frac{1}{1 - 9x}$$

Which of the following is the power series representation for g centered at 0?

- **A.** $-\frac{1}{9}\sum_{k=1}^{\infty}\frac{(9x)^k}{k}$
- **C.** $-9\sum_{k=1}^{\infty}\frac{(9x)^k}{k}$

The interval of convergence is __________.
(Simplify your answer. Type your answer in interval notation.)

Answers D. $-\sum_{k=1}^{\infty}\frac{(9x)^k}{k}$

$\left[-\frac{1}{9},\frac{1}{9}\right)$

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3. Find the interval of convergence of the series.

$$\sum_{n=0}^{\infty} \frac{(x-4)^n}{n^2 6^n}$$

- A. $3 \le x \le 5$
- B. $-10 < x < 10$
- C. $x < 10$
- D. $-2 \le x \le 10$

Answer: D. $-2 \le x \le 10$

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4. For the following telescoping series, find a formula for the nth term of the sequence of partial sums $\{S_n\}$. Then evaluate $\lim_{n\to\infty} S_n$ to obtain the value of the series or state that the series diverges.

$$\sum_{k=1}^{\infty} \frac{16}{(4k-1)(4k+3)}$$

$S_n =$ __________

Select the correct choice and fill in any answer boxes in your choice below.

- A. $\sum_{k=1}^{\infty} \frac{16}{(4k-1)(4k+3)} =$ __________ (Simplify your answer.)
- B. The series diverges.

Answers $\frac{4}{3} - \frac{4}{4n+3}$

A. $\sum_{k=1}^{\infty} \frac{16}{(4k-1)(4k+3)} = \mathbf{\frac{4}{3}}$ (Simplify your answer.)

5. Find the Taylor polynomials p_1, ..., p_4 centered at a = 0 for f(x) = **cos** (− 2x).

$p_1(x) =$ ____________

$p_2(x) =$ ____________

$p_3(x) =$ ____________

$p_4(x) =$ ____________

Answers 1

$1 - 2x^2$

$1 - 2x^2$

$1 - 2x^2 + \frac{2}{3}x^4$

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6. Use the Ratio Test to determine if the series converges.

$$\sum_{k=1}^{\infty} \frac{6(k!)^2}{7(2k)!}$$

Select the correct choice below and fill in the answer box to complete your choice.

- **A.** The series diverges because r = ____________.
- **B.** The series converges because r = ____________.
- **C.** The Ratio Test is inconclusive because r = ____________.

Answer: B. The series converges because r = $\mathbf{\frac{1}{4}}$.

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7. **a.** Find the nth-order Taylor polynomials of the given function centered at the given point a, for n = 0, 1, and 2.
b. Graph the Taylor polynomials and the function.

$$f(x) = \sin x,\ a = \frac{3\pi}{4}$$

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a. Find the Taylor polynomial of order 0. Choose the correct answer below.

- A. $p_0(x) = \frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right)$
- B. $p_0(x) = \frac{\sqrt{2}}{2}$
- C. $p_0(x) = 0$
- D. $p_0(x) = 1$

Find the Taylor polynomial of order 1.

- A. $p_1(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right)$
- B. $p_1(x) = \frac{\sqrt{2}}{2}$
- C. $p_1(x) = \frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right) - \frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right)^2$
- D. $p_1(x) = \left(x - \frac{3\pi}{4}\right)$

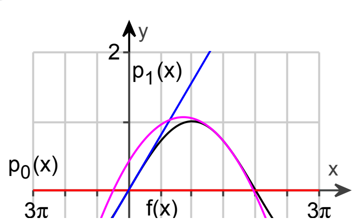
Find the Taylor polynomial of order 2.

- A. $p_2(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{3\pi}{4}\right)^2$
- B. $p_2(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4}\left(x - \frac{3\pi}{4}\right)$
- C. $p_2(x) = \frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right) - \frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right)^2 - \frac{\sqrt{2}}{4}\left(x - \frac{3\pi}{4}\right)^3$
- D. $p_2(x) = \left(x - \frac{3\pi}{4}\right) - \frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right)^2$

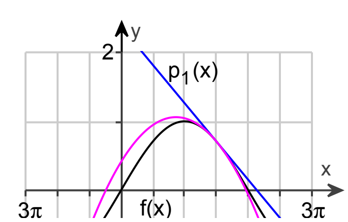
b. Choose the correct graph below.

$$f(x) = \sin x,\ a = \frac{3\pi}{4}$$

- A.



- B.

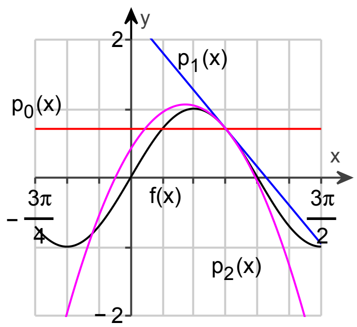


Answers

B. $p_0(x) = \frac{\sqrt{2}}{2}$

A. $p_1(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right)$

A. $p_2(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{3\pi}{4}\right)^2$



C.

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8. Use the Comparison Test or the Limit Comparison Test to determine whether the following series converges.

$$\sum_{n=1}^{\infty} \frac{1}{9\sqrt{n} + \sqrt[3]{n}}$$

Choose the correct answer below.

- **A.** The Limit Comparison Test with $\frac{1}{\sqrt{n}}$ shows that the series converges.
- **B.** The Comparison Test with $\sqrt{n}$ shows that the series diverges.
- **C.** The Limit Comparison Test with $\frac{1}{\sqrt[3]{n}}$ shows that the series converges.
- **D.** The Comparison Test with $\sqrt[3]{n}$ shows that the series converges.
- **E.** The Limit Comparison Test with $\frac{1}{\sqrt[3]{n}}$ shows that the series diverges.
- **F.** The Limit Comparison Test with $\frac{1}{\sqrt{n}}$ shows that the series diverges.

Answer:
F. The Limit Comparison Test with $\frac{1}{\sqrt{n}}$ shows that the series diverges.

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9. Use the Root Test to determine whether the series converges.

$$\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{3k^2}$$

Select the correct choice below and fill in the answer box to complete your choice.
(Type an exact answer in terms of e.)

- **A.** The series converges because $\rho =$ ________.
- **B.** The series diverges because $\rho =$ ________.
- **C.** The Root Test is inconclusive because $\rho =$ ________.

Answer: A. The series converges because $\rho = \frac{1}{e^3}$.

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10. Evaluate the series or state that it diverges.

$$\sum_{k=1}^{\infty} \left[\frac{2}{5}\left(\frac{1}{7}\right)^k + \frac{3}{5}\left(\frac{7}{9}\right)^k\right]$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $\sum_{k=1}^{\infty} \left[\frac{2}{5}\left(\frac{1}{7}\right)^k + \frac{3}{5}\left(\frac{7}{9}\right)^k\right] =$ ________ (Simplify your answer.)
- **B.** The series diverges.

Answer: A. $\sum_{k=1}^{\infty} \left[\frac{2}{5}\left(\frac{1}{7}\right)^k + \frac{3}{5}\left(\frac{7}{9}\right)^k\right] = \frac{13}{6}$ (Simplify your answer.)

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11. Use the Divergence Test to determine whether the following series diverges or state that the test is inconclusive.

$$\sum_{k=1}^{\infty} \frac{7k^2}{k!}$$

Choose the correct answer below.

- A. The series converges because $\lim_{k\to\infty} \frac{7k^2}{k!} \neq 0$.
- B. The series diverges because $\lim_{k\to\infty} \frac{7k^2}{k!} = 0$.
- C. The series converges because $\lim_{k\to\infty} \frac{7k^2}{k!} = 0$.
- D. The series diverges because $\lim_{k\to\infty} \frac{7k^2}{k!} \neq 0$.
- E. The Divergence Test is inconclusive.

Answer: E. The Divergence Test is inconclusive.

12. Use the Integral Test to determine whether the following series converges after showing that the conditions of the Integral Test are satisfied.

$$\sum_{k=1}^{\infty} \frac{2e^{k}}{1+e^{2k}}$$

Determine which of the necessary properties of the function that will be used for the Integral Test has. Select all that apply.

- ☐ **A.** The function f(x) is a decreasing function for $x \geq 1$.
- ☐ **B.** The function f(x) is an increasing function for $x \geq 1$.
- ☐ **C.** The function f(x) is continuous for $x \geq 1$.
- ☐ **D.** The function f(x) has the property that $a_k = f(k)$ for k = 1, 2, 3, ...
- ☐ **E.** The function f(x) is negative for $x \geq 1$.
- ☐ **F.** The function f(x) is positive for $x \geq 1$.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ○ **A.** The series converges. The value of the integral $\int_1^{\infty} \frac{2e^{x}}{1+e^{2x}}\,dx$ is ________.
  (Type an exact answer.)
- ○ **B.** The series diverges. The value of the integral $\int_1^{\infty} \frac{2e^{x}}{1+e^{2x}}\,dx$ is ________.
  (Type an exact answer.)
- ○ **C.** The Integral Test does not apply to this series.

Answers A. The function f(x) is a decreasing function for $x \geq 1$., C. The function f(x) is continuous for $x \geq 1$., D. The function f(x) has the property that $a_k = f(k)$ for k = 1, 2, 3, ..., F. The function f(x) is positive for $x \geq 1$.

A. The series converges. The value of the integral $\int_1^{\infty} \frac{2e^{x}}{1+e^{2x}}\,dx$ is $\mathbf{2\left(\frac{\pi}{2} - \tan^{-1} e\right)}$.

(Type an exact answer.)