1. Determine whether the following series converges absolutely, converges conditionally, or diverges.

\[ \sum_{k=1}^{\infty} \frac{(-1)^k}{k^{12} + 16} \]

Find \( \lim_{k \to \infty} a_k \). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. \( \lim_{k \to \infty} a_k = \) ____________

B. The limit does not exist.

Now, let \( \sum a_k \) denote \( \sum_{k=1}^{\infty} \frac{(-1)^k k^5}{\sqrt{k^{12} + 16}} \). What can be concluded from this result using the Divergence Test?

A. The series \( \sum a_k \) must diverge.
B. The series \( \sum |a_k| \) must converge.
C. The series \( \sum |a_k| \) must diverge.
D. The series \( \sum a_k \) must converge.
E. The Divergence Test is inconclusive.

Are the terms of the sequence \( |a_k| \) decreasing after some point?

yes
no

Let \( \sum a_k \) denote \( \sum_{k=1}^{\infty} \frac{(-1)^k k^5}{\sqrt{k^{12} + 16}} \). What can be concluded from these results using the Alternating Series Test?

A. The series \( k^6 \) must diverge.
B. The series \( \sum a_k \) must diverge.
C. The series \( k^6 \) must converge.
D. The series \( \sum a_k \) must converge.
E. The Alternating Series Test does not apply to this series.

Does the series \( \sum |a_k| \) converge?

A. yes, as can be determined by the Limit Comparison Test
B. no, as can be determined by the Limit Comparison Test
C. no, because of the Divergence Test
Does the series \( \sum a_k \) converge absolutely, converge conditionally, or diverge?

A. The series converges conditionally because \( \sum |a_k| \) converges but \( \sum a_k \) diverges.

B. The series diverges because \( \sum |a_k| \) diverges.

C. The series converges conditionally because \( \sum a_k \) converges but \( \sum |a_k| \) diverges.

D. The series converges absolutely because \( \sum |a_k| \) converges.

E. The series diverges because \( \lim_{k \to \infty} a_k \neq 0 \).

Answers

A. \( \lim_{k \to \infty} a_k = 0 \)

E. The Divergence Test is inconclusive.

yes

D. The series \( \sum a_k \) must converge.

B. no, as can be determined by the Limit Comparison Test

C. The series converges conditionally because \( \sum a_k \) converges but \( \sum |a_k| \) diverges.

2. Find the power series representation for \( g \) centered at 0 by differentiating or integrating the power series for \( f \) (perhaps more than once). Give the interval of convergence for the resulting series.

\[ g(x) = \ln(1 - 9x) \text{ using } f(x) = \frac{1}{1 - 9x} \]

Which of the following is the power series representation for \( g \) centered at 0?

A. \[ -\frac{1}{9} \sum_{k=1}^{\infty} \frac{(9x)^k}{k} \]

C. \[ -9 \sum_{k=1}^{\infty} \frac{(9x)^k}{k} \]

The interval of convergence is \( \left( -\frac{1}{9}, \frac{1}{9} \right) \).

(Simplify your answer. Type your answer in interval notation.)

Answers

D. \[ -\sum_{k=1}^{\infty} \frac{(9x)^k}{k} \left( -\frac{1}{9}, \frac{1}{9} \right) \]
3. Find the interval of convergence of the series.

\[ \sum_{n=0}^{\infty} \frac{(x - 4)^n}{n^2 6^n} \]

- A. \( 3 \leq x \leq 5 \)
- B. \( -10 < x < 10 \)
- C. \( x < 10 \)
- D. \( -2 \leq x \leq 10 \)

Answer: D. \( -2 \leq x \leq 10 \)

4. For the following telescoping series, find a formula for the nth term of the sequence of partial sums \( \{S_n\} \). Then evaluate \( \lim_{n \to \infty} S_n \) to obtain the value of the series or state that the series diverges.

\[ \sum_{k=1}^{\infty} \frac{16}{(4k-1)(4k+3)} \]

\( S_n = \) __________

Select the correct choice and fill in any answer boxes in your choice below.

- A. \( \sum_{k=1}^{\infty} \frac{16}{(4k-1)(4k+3)} = \) __________ (Simplify your answer.)
- B. The series diverges.

Answers: \( \frac{4}{3} - \frac{4}{4n+3} \)

A. \( \sum_{k=1}^{\infty} \frac{16}{(4k-1)(4k+3)} = \frac{4}{3} \) (Simplify your answer.)
5. Find the Taylor polynomials \( p_1, \ldots, p_4 \) centered at \( a = 0 \) for \( f(x) = \cos(-2x) \).

\[
p_1(x) = \quad \quad \\
p_2(x) = \quad \quad \\
p_3(x) = \quad \quad \\
p_4(x) = \quad \quad \\
\]

Answers
\[
1 - 2x^2 \\
1 - 2x^2 \\
1 - 2x^2 + \frac{2}{3}x^4 \\
\]

6. Use the Ratio Test to determine if the series converges.

\[
\sum_{k=1}^{\infty} \frac{6(k!)^2}{7(2k)!} 
\]

Select the correct choice below and fill in the answer box to complete your choice.

- **A.** The series diverges because \( r = \quad \quad \).
- **B.** The series converges because \( r = \quad \quad \).
- **C.** The Ratio Test is inconclusive because \( r = \quad \quad \).

Answer: B. The series converges because \( r = \frac{1}{4} \).
7. a. Find the nth-order Taylor polynomials of the given function centered at the given point \( a \), for \( n = 0, 1, \) and 2.

\[ f(x) = \sin x, \quad a = \frac{3\pi}{4} \]

b. Graph the Taylor polynomials and the function.

a. Find the Taylor polynomial of order 0. Choose the correct answer below.

- A. \( p_0(x) = \frac{\sqrt{2}}{2} \left( x - \frac{3\pi}{4} \right) \)
- B. \( p_0(x) = \frac{\sqrt{2}}{2} \)
- C. \( p_0(x) = 0 \)
- D. \( p_0(x) = 1 \)

Find the Taylor polynomial of order 1.

- A. \( p_1(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \left( x - \frac{3\pi}{4} \right) \)
- B. \( p_1(x) = \frac{\sqrt{2}}{2} \)
- C. \( p_1(x) = \frac{\sqrt{2}}{2} \left( x - \frac{3\pi}{4} \right) - \frac{\sqrt{2}}{2} \left( x - \frac{3\pi}{4} \right)^2 \)
- D. \( p_1(x) = \left( x - \frac{3\pi}{4} \right) \)

Find the Taylor polynomial of order 2.

- A. \( p_2(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \left( x - \frac{3\pi}{4} \right) - \frac{\sqrt{2}}{4} \left( x - \frac{3\pi}{4} \right)^2 \)
- B. \( p_2(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} \left( x - \frac{3\pi}{4} \right) \)
- C. \( p_2(x) = \frac{\sqrt{2}}{2} \left( x - \frac{3\pi}{4} \right) - \frac{\sqrt{2}}{2} \left( x - \frac{3\pi}{4} \right)^2 - \frac{\sqrt{2}}{4} \left( x - \frac{3\pi}{4} \right)^3 \)
- D. \( p_2(x) = \left( x - \frac{3\pi}{4} \right) - \frac{\sqrt{2}}{2} \left( x - \frac{3\pi}{4} \right)^2 \)

b. Choose the correct graph below.

\[ f(x) = \sin x, \quad a = \frac{3\pi}{4} \]

- A.
- B.
8. Use the Comparison Test or the Limit Comparison Test to determine whether the following series converges.

\[ \sum_{n=1}^{\infty} \frac{1}{9\sqrt{n} + 3\sqrt{n}} \]

Choose the correct answer below.

- **A.** The Limit Comparison Test with \( \frac{1}{\sqrt{n}} \) shows that the series converges.
- **B.** The Comparison Test with \( \sqrt{n} \) shows that the series diverges.
- **C.** The Limit Comparison Test with \( \frac{1}{3\sqrt{n}} \) shows that the series converges.
- **D.** The Comparison Test with \( \frac{3}{\sqrt{n}} \) shows that the series converges.
- **E.** The Limit Comparison Test with \( \frac{1}{3\sqrt{n}} \) shows that the series diverges.
- **F.** The Limit Comparison Test with \( \frac{1}{\sqrt{n}} \) shows that the series diverges.

**Answer:**
F. The Limit Comparison Test with \( \frac{1}{\sqrt{n}} \) shows that the series diverges.
9. Use the Root Test to determine whether the series converges.

\[ \sum_{k=1}^{\infty} \left( \frac{k}{k+1} \right)^{3k^2} \]

Select the correct choice below and fill in the answer box to complete your choice.
(Type an exact answer in terms of e.)

- **A.** The series converges because \( \rho = \frac{1}{e^3} \).
- **B.** The series diverges because \( \rho = \) \( \) .
- **C.** The Root Test is inconclusive because \( \rho = \) \( \) .

Answer: A. The series converges because \( \rho = \frac{1}{e^3} \).

10. Evaluate the series or state that it diverges.

\[ \sum_{k=1}^{\infty} \left[ \frac{2}{5} \left( \frac{1}{7} \right)^k + \frac{3}{5} \left( \frac{7}{9} \right)^k \right] \]

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** \( \sum_{k=1}^{\infty} \left[ \frac{2}{5} \left( \frac{1}{7} \right)^k + \frac{3}{5} \left( \frac{7}{9} \right)^k \right] = \) \( \) \( \) (Simplify your answer.)
- **B.** The series diverges.

Answer: A. \( \sum_{k=1}^{\infty} \left[ \frac{2}{5} \left( \frac{1}{7} \right)^k + \frac{3}{5} \left( \frac{7}{9} \right)^k \right] = \frac{13}{6} \) (Simplify your answer.)
11. Use the Divergence Test to determine whether the following series diverges or state that the test is inconclusive.

\[ \sum_{k=1}^{\infty} \frac{7k^2}{k!} \]

Choose the correct answer below.

- **A.** The series converges because \( \lim_{k \to \infty} \frac{7k^2}{k!} \neq 0 \).
- **B.** The series diverges because \( \lim_{k \to \infty} \frac{7k^2}{k!} = 0 \).
- **C.** The series converges because \( \lim_{k \to \infty} \frac{7k^2}{k!} = 0 \).
- **D.** The series diverges because \( \lim_{k \to \infty} \frac{7k^2}{k!} \neq 0 \).
- **E.** The Divergence Test is inconclusive.

Answer: E. The Divergence Test is inconclusive.
12. Use the Integral Test to determine whether the following series converges after showing that the conditions of the Integral Test are satisfied.

\[ \sum_{k=1}^{\infty} \frac{2e^k}{1 + e^{2k}} \]

Determine which of the necessary properties of the function that will be used for the Integral Test has. Select all that apply.

- [ ] A. The function \( f(x) \) is a decreasing function for \( x \geq 1 \).
- [ ] B. The function \( f(x) \) is an increasing function for \( x \geq 1 \).
- [ ] C. The function \( f(x) \) is continuous for \( x \geq 1 \).
- [ ] D. The function \( f(x) \) has the property that \( a_k = f(k) \) for \( k = 1, 2, 3, \ldots \).
- [ ] E. The function \( f(x) \) is negative for \( x \geq 1 \).
- [ ] F. The function \( f(x) \) is positive for \( x \geq 1 \).

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- [ ] A. The series converges. The value of the integral \( \int_{1}^{\infty} \frac{2e^x}{1 + e^{2x}} \, dx \) is ____________________.
  (Type an exact answer.)

- [ ] B. The series diverges. The value of the integral \( \int_{1}^{\infty} \frac{2e^x}{1 + e^{2x}} \, dx \) is ____________________.
  (Type an exact answer.)

- [ ] C. The Integral Test does not apply to this series.

Answers:

- A. The function \( f(x) \) is a decreasing function for \( x \geq 1 \).
- C. The function \( f(x) \) is continuous for \( x \geq 1 \).
- D. The function \( f(x) \) has the property that \( a_k = f(k) \) for \( k = 1, 2, 3, \ldots \).
- F. The function \( f(x) \) is positive for \( x \geq 1 \).

A. The series converges. The value of the integral \( \int_{1}^{\infty} \frac{2e^x}{1 + e^{2x}} \, dx \) is \( 2 \left( \frac{\pi}{2} - \tan^{-1} e \right) \).

(Type an exact answer.)