Student: ____________________ **Date:** ____________________	**Instructor:** Tong Ding, Purdue Math, Jennifer Hobbs, Stephanie Foster **Course:** MA166-123- Analytic Geom. And Calc. II-Spring 2020	**Assignment:** FinalExam

1. Evaluate the following integral using trigonometric substitution.

$$\int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx$$

What substitution will be the most helpful for evaluating this integral?

- **A.** $x = \sin\theta$
- **B.** $x = \sec\theta$
- **C.** $x = \tan\theta$

Rewrite the given integral using this substitution.

$$\int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx = \int_0^{\_\_\_\_\_} (\_\_\_\_\_\_)\,d\theta$$

(Simplify your answers. Type exact answers.)

Evaluate the integral.

$$\int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx = \_\_\_\_\_\_ \text{ (Type an exact answer.)}$$

Answers A. $x = \sin\theta$

$\frac{\pi}{4}$

$\sin^2\theta$

$\frac{\pi}{8} - \frac{1}{4}$

2. Evaluate the following integral.

$$\int 3\sin^3 x \cos^2 x\,dx$$

$$\int 3\sin^3 x \cos^2 x\,dx = \_\_\_\_\_\_$$

Answer: $\frac{3}{5}\cos^5 x - \cos^3 x + C$

3. Let R be the region bounded by the following curves. Find the volume of the solid generated when R is revolved about the x-axis.

$y = 4|x|$, $y = 5 - x^2$

The volume of the solid is __________ cubic units.
(Type an exact answer.)

Answer: $\frac{496\pi}{15}$

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4. Find the area of the parallelogram that has adjacent sides $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{v} = 2\mathbf{j} - 3\mathbf{k}$.

The area of the parallelogram is __________.
(Type an exact answer, using radicals as needed.)

Answer: $\sqrt{61}$

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5. Find the slope of the line tangent to the polar curve at the given point.

$r = 5\,\mathbf{sin}\,\theta$; $\left(\frac{5}{2}, \frac{\pi}{6}\right)$

Find $\frac{dy}{dx}$ as a function of θ.

$\frac{dy}{dx} =$ __________

Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** The slope of the curve at the point $\left(\frac{5}{2}, \frac{\pi}{6}\right)$ is __________. (Type an exact answer.)
- **B.** The slope of the curve at the point $\left(\frac{5}{2}, \frac{\pi}{6}\right)$ is undefined.

Answers $\frac{2\,\mathbf{cos}\,\theta\,\mathbf{sin}\,\theta}{\mathbf{cos}^2\theta - \mathbf{sin}^2\theta}$

A. The slope of the curve at the point $\left(\frac{5}{2}, \frac{\pi}{6}\right)$ is $\sqrt{3}$. (Type an exact answer.)

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6. Use the general slicing method to find the volume of the following solid.

The solid whose base is the triangle with vertices (0,0), (10,0), and (0,10) and whose cross sections perpendicular to the base and parallel to the y-axis are semicircles

Set up the integral that gives the volume of the solid. Use increasing limits of integration. Select the correct choice below and fill in the answer boxes to complete your choice.

(Type exact answers.)

- A. $\int_{\_\_\_}^{\_\_\_} (\_\_\_\_)\,dx$
- B. $\int_{\_\_\_}^{\_\_\_} (\_\_\_\_)\,dy$

The volume of the solid is ______ cubic units. (Type an exact answer.)

Answers A. $\int_{0}^{10} \left(\frac{1}{8}\pi(10-x)^2\right) dx$

$\frac{125\pi}{3}$

7. Use the Divergence Test to determine whether the following series diverges or state that the test is inconclusive.

$$\sum_{n=0}^{\infty} \frac{n^3}{6n^3+1}$$

Select the correct answer below and fill in the answer box to complete your choice.

- A. According to the Divergence Test, the series diverges because $\lim_{k\to\infty} a_k =$ ______.
  (Simplify your answer.)
- B. According to the Divergence Test, the series converges because $\lim_{k\to\infty} a_k =$ ______.
  (Simplify your answer.)
- C. The Divergence Test is inconclusive because $\lim_{k\to\infty} a_k =$ ______.
  (Simplify your answer.)
- D. The Divergence Test is inconclusive because $\lim_{k\to\infty} a_k$ does not exist.

Answer: A. According to the Divergence Test, the series diverges because $\lim_{k\to\infty} a_k = \frac{1}{6}$.

(Simplify your answer.)

8. For the vectors $\mathbf{u} = \langle -3,0,2 \rangle$ and $\mathbf{v} = \langle 1,4,-4 \rangle$, calculate $\text{proj}_{\mathbf{v}}\mathbf{u}$ and $\text{scal}_{\mathbf{v}}\mathbf{u}$.

$\text{proj}_{\mathbf{v}}\mathbf{u} = \langle$ ________, ________, ________ $\rangle$

$\text{scal}_{\mathbf{v}}\mathbf{u} =$ ________

(Type an exact answer, using radicals as needed.)

Answers $-\frac{1}{3}$

$-\frac{4}{3}$

$\frac{4}{3}$

$-\frac{\sqrt{33}}{3}$

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9. Evaluate the following integral using integration by parts.

$$\int 9x\, e^{2x}\, dx$$

Evaluate the integral.

$\int 9x\, e^{2x}\, dx =$ ________

Answer: $\frac{9}{2} x e^{2x} - \frac{9}{4} e^{2x} + C$

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10. Evaluate the following integral using trigonometric substitution.

$$\int \frac{\sqrt{361-x^2}}{x}\,dx$$

What substitution will be the most helpful for evaluating this integral?

- **A.** $x = 19\,\mathbf{sec}\,\theta$
- **B.** $x = 19\,\mathbf{tan}\,\theta$
- **C.** $x = 19\,\mathbf{sin}\,\theta$

Rewrite the given integral using this substitution.

$$\int \frac{\sqrt{361-x^2}}{x}\,dx = \int \left(\_\_\_\_\_\_\_\_\_\_\right) d\theta$$

(Simplify your answers. Type exact answers.)

Evaluate the integral.

$$\int \frac{\sqrt{361-x^2}}{x}\,dx = \_\_\_\_\_\_\_\_\_\_$$

(Type an exact answer.)

Answers C. $x = 19\,\mathbf{sin}\,\theta$

$19(\mathbf{csc}\;\theta - \mathbf{sin}\;\theta)$

$$19\,\mathbf{ln}\left|\frac{\sqrt{361-x^2}-19}{x}\right| + \sqrt{361-x^2} + C$$

11. Determine whether the following series converges.

$$\sum_{k=1}^{\infty} (-1)^k k \sin \frac{1}{k}$$

Let $a_k \geq 0$ represent the magnitude of the terms of the given series. Select the correct choice below and fill in the answer box(es) to complete your choice.

- **A.** The series diverges because a_k = ____________ is nondecreasing in magnitude for k greater than some index N a
- **B.** The series converges because a_k = ____________ is nondecreasing in magnitude for k greater than some index N
- **C.** The series converges because a_k = ____________ is nonincreasing in magnitude for k greater than some index N.
- **D.** The series diverges because a_k = ____________ and for any index N, there are some values of k > N for which a_k
- **E.** The series diverges because a_k = ____________ is nonincreasing in magnitude for k greater than some index N.
- **F.** The series converges because a_k = ____________ and for any index N, there are some values of k > N for which a

Answer: A.

The series diverges because $a_k = \mathbf{k \sin \frac{1}{k}}$ is nondecreasing in magnitude for k greater than some index N and $\lim_{k\to\infty} a_k = \mathbf{1}$.

12. Find the area of the surface generated when the given curve is revolved about the x-axis.

$$y = \frac{x^3}{3} + \frac{1}{4x}, \text{ for } \frac{1}{2} \leq x \leq 2$$

The area of the surface is ____________ square units.
(Type an exact answer, using π as needed.)

Answer: $\frac{275}{32}\pi$

13. Integrate the given function.

$$\int 3 \cos^2 x \, dx$$

$\int 3 \cos^2 x \, dx =$ ____________

Answer: $\frac{3}{2}x + \frac{3}{4} \sin 2x + C$

14. Find the equation of the sphere passing through P(− 6, 1, 4) and Q(4, − 3, 3) with its center at the midpoint of PQ.

The standard equation of the sphere is ____________.
(Simplify your answer.)

Answer: $(x+1)^2 + (y+1)^2 + \left(z - \frac{7}{2}\right)^2 = \frac{117}{4}$

15. A 40-m-long chain hangs vertically from a cylinder attached to a winch. Assume there is no friction in the system and that the chain has a density of 10 kg/m. Use 9.8 m/s^2 for the acceleration due to gravity.

a. How much work is required to wind the entire chain onto the cylinder using the winch?
b. How much work is required to wind the chain onto the cylinder if a 25-kg block is attached to the end of the chain?

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a. Set up the integral that gives the work required to wind the entire chain onto the cylinder using the winch. Use increasing limits of integration.

$$\int_{\_\_\_}^{\_\_\_} (\_\_\_\_\_\_)\, dy$$

(Type exact answers.)

The amount of work required is ______ (1) ______
(Type an integer or a decimal.)

b. Set up the integral that gives the work required to wind the chain onto the cylinder if a 25-kg block is attached to the end of the chain. Use increasing limits of integration.

$$\int_{\_\_\_}^{\_\_\_} (\_\_\_\_\_\_)\, dy$$

(Type exact answers.)

The amount of work required if a 25-kg block is attached to the end of the chain is ______ (2) ______
(Type an integer or a decimal.)

(1) ○ kg / m.
○ m.
○ m/s^2.
○ J.

(2) ○ kg / m.
○ m.
○ J.
○ m/s^2.

Answers 0

40

$9.8(400 - 10y)$

78,400

(1) J.

0

40

$9.8(400 - 10y) + 25 \cdot 9.8$

88,200

(2) J.

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16. Use the Integral Test to determine the convergence or divergence of the following series, or state that the test does not apply.

$$\sum_{k=4}^{\infty} \frac{5}{k(\ln k)^2}$$

Select the correct choice below and, if necessary, fill in the answer box to complete the choice.

- A. The series converges. The value of the integral $\int_4^{\infty} \frac{5}{x(\ln x)^2}dx$ is ________.
  (Type an exact answer.)
- B. The series diverges. The value of the integral $\int_4^{\infty} \frac{5}{x(\ln x)^2}dx$ is ________.
  (Type an exact answer.)
- C. The Integral Test does not apply.

Answer: A. The series converges. The value of the integral $\int_4^{\infty} \frac{5}{x(\ln x)^2}dx$ is $\frac{5}{\ln 4}$.(Type an exact answer.)

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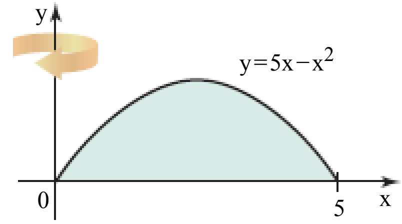
17. Find the length of the cardioid $r = 1 + \cos\theta$.

The length of the cardioid is ________.
(Simplify your answer.)

Answer: 8

18. Let R be the region bounded by the following curves. Use the shell method to find the volume of the solid generated when R is revolved about the y-axis.

$y = 5x - x^2,\ y = 0$



Set up the integral that gives the volume of the solid using the shell method. Use increasing limits of integration. Select the correct choice below and fill in the answer boxes to complete your choice.

(Type exact answers.)

○ **A.** $\int_{\_\_\_}^{\_\_\_} (\_\_\_\_\_\_)\, dy$

○ **B.** $\int_{\_\_\_}^{\_\_\_} (\_\_\_\_\_\_)\, dx$

The volume is __________. (Type an exact answer.)

Answers B. $\int_{0}^{5} \left(2\pi x\left(5x - x^2\right)\right) dx$

$\frac{625\pi}{6}$

19. Replace x by $\frac{x}{4} - 1$ in the series $\ln (1 + x)^3 = 3 \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}$ to obtain a power series for $\ln \left(\frac{x}{4}\right)^3$ centered at x = 4.
What is the interval of convergence for the new power series?

Choose the correct power series below.

○ **A.** $3 \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-1)^k}{4^k k}$

○ **C.** $3 \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-1)^k}{k}$

The interval of convergence is __________.
(Simplify your answer. Type your answer in interval notation.)

Answers D. $3 \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-4)^k}{4^k k}$

(0,8]

20. Convert the equation $r = -18 \sin\theta$ to Cartesian coordinates. Describe the resulting curve.

Choose the correct equation below.

- A. $(x-9)^2 + y^2 = 81$
- B. $(x+9)^2 + y^2 = 81$
- C. $x^2 + (y-9)^2 = 81$
- D. $x^2 + (y+9)^2 = 81$

The equation describes a circle.

The center of the circle is ______. (Simplify your answer. Type an ordered pair.)

The radius is ______. (Simplify your answer.)

Answers D. $x^2 + (y+9)^2 = 81$

$(0, -9)$

9

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21. Convert the following equation to Cartesian coordinates. Describe the resulting curve.

$$r = \frac{8}{2\cos\theta + 5\sin\theta}$$

Write the Cartesian equation.

______

Describe the curve. Select the correct choice below and, if necessary, fill in any answer box(es) to complete your choice.

- A. The curve is a circle centered at the point ______ with radius ______.
(Type exact answers, using radicals as needed.)
- B. The curve is a line with slope ______ and a y-intercept at the point ______.
(Type exact answers, using radicals as needed.)
- C. The curve is a vertical line with x-intercept at the point ______.
(Type exact answers, using radicals as needed.)

Answers $y = -\frac{2}{5}x + \frac{8}{5}$

B. The curve is a line with slope $-\frac{2}{5}$ and a y-intercept at the point $\left(0, \frac{8}{5}\right)$.

(Type exact answers, using radicals as needed.)

22. Make a sketch of the region and its bounding curves. Find the area of the region.

The region inside one leaf of $r = 3\cos 5\theta$

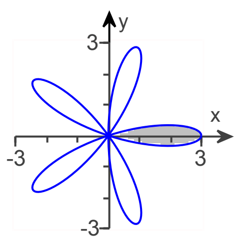
Choose the correct graph of the region below.

A.

B.

The area of the region is ________ square units. (Type an exact answer, using π as needed.)

Answers



A.

$\frac{9\pi}{20}$

23. Evaluate the integral or state that it diverges.

$$\int_4^{\infty} \frac{x}{(x+3)^2}\,dx$$

Select the correct choice and, if necessary, fill in the answer box to complete your choice.

A. The integral converges to ________.

B. The integral diverges.

Answer: B. The integral diverges.

24. Evaluate the following definite integral.

$$\int_0^1 \frac{12}{x^2-9}\,dx$$

Find the partial fraction decomposition of the integrand.

$$\int_0^1 \frac{12}{x^2-9}\,dx = \int_0^1 \left(\underline{\qquad\qquad}\right) dx$$

Evaluate the definite integral.

$$\int_0^1 \frac{12}{x^2-9}\,dx = \underline{\qquad\qquad}$$

(Type an exact answer.)

Answers $-\frac{2}{x+3}+\frac{2}{x-3}$

$\ln \frac{1}{4}$

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25. Determine whether the following series converge. Justify your answer.

$$\frac{23}{2\cdot 3}+\frac{23}{4\cdot 5}+\frac{23}{6\cdot 7}+\frac{23}{8\cdot 9}+\ldots$$

Select the correct choice below and fill in the answer box to complete your choice.
(Type an exact answer.)

- **A.** Let $a_k = \frac{23}{4k^2+2k}$ and $b_k = \frac{1}{k^2}$. Since $\lim_{k\to\infty} \frac{a_k}{b_k} = \underline{\qquad\qquad}$, the series converges by the Limit Comparison Te
- **B.** Let $a_k = \frac{23}{4k^2+2k}$ and $b_k = \frac{1}{k^2}$. Since $\lim_{k\to\infty} \frac{a_k}{b_k} = \underline{\qquad\qquad}$, the series diverges by the Limit Comparison Test.
- **C.** The Ratio Test yields r = $\underline{\qquad\qquad}$. This is greater than 1, so the series diverges by the Ratio Test.
- **D.** The limit of the terms of the series is $\underline{\qquad\qquad}$, so the series converges by the Divergence Test.
- **E.** The Ratio Test yields r = $\underline{\qquad\qquad}$. This is less than 1, so the series converges by the Ratio Test.

Answer: A.

Let $a_k = \frac{23}{4k^2+2k}$ and $b_k = \frac{1}{k^2}$. Since $\lim_{k\to\infty} \frac{a_k}{b_k} = \mathbf{\frac{23}{4}}$, the series converges by the Limit Comparison Test.