Answers to Exam 2 (Green)

1. **B**

2. **E**

3. **D**

4. **C**

5. **A**

6. **D**

7. **E**

8. (1) The image of $p(t) = 1 + 2t + 3t^2$ is $T(p(t)) = 1 + t + 5t^2$

   (2) The pre-image of $p(t) = 1 + 2t + 3t^2$ is $1 + t + t^2$

   (3) $[T]_\mathcal{B} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -1 & 0 \\ 0 & 4 & -1 \end{bmatrix}$

9. (1) $\lambda_1 = -1$, a basis is $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$ $\quad \lambda_2 = \lambda_3 = -3$, a basis is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \right\}$
   Answer may vary!

   (2) $P = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & -1 \\ 1 & 0 & 3 \end{bmatrix} \quad D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ Answer may vary!

10. (1) $\lambda_1 = -1$, $\lambda_2 = 9$.

    (2) $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1 \begin{bmatrix} 3e^{-t} \\ -4e^{-t} \end{bmatrix} + c_2 \begin{bmatrix} e^{9t} \\ 2e^{9t} \end{bmatrix}$ for arbitrary constants c_1 and c_2. Answer may vary!

    (3) $c_1 = 2, c_2 = 3, x(1) + y(1) = -2e^{-1} + 9e^9$.