

MA 15400

Fall 2013

Exam 1

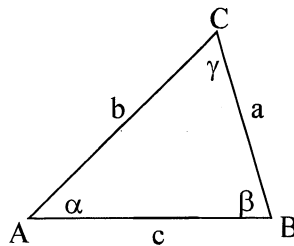
Solutions

PYTHAGOREAN IDENTITIES:

$$\sin^2 \theta + \cos^2 \theta = 1$$

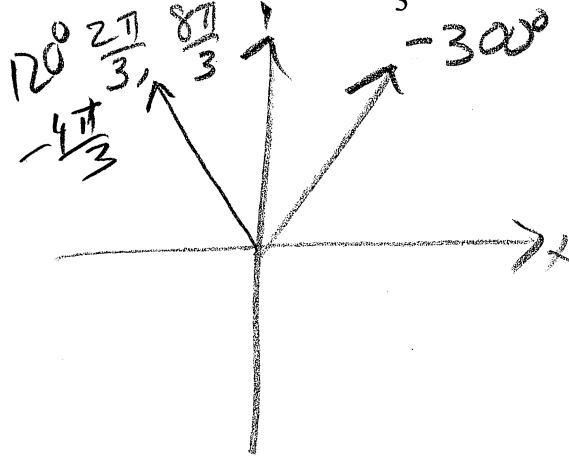
$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$



1. Which of the angles is **not** coterminal with $\frac{2\pi}{3}$?

- A. $\frac{8\pi}{3}$
- B. -300°**
- C. 840°
- D. $-\frac{4\pi}{3}$
- E. $\frac{14\pi}{3}$



$$\begin{array}{r} 840^\circ \\ - 360^\circ \\ \hline 480^\circ \\ - 360^\circ \\ \hline 120^\circ \end{array} \qquad \begin{array}{r} \frac{14\pi}{3} - \frac{6\pi}{3} \\ = \frac{8\pi}{3} - \frac{6\pi}{3} \\ = \frac{2\pi}{3} \end{array}$$

2. Express the angle $200^\circ 31' 9''$ as a decimal, to the nearest ten-thousandth of a degree.

- A. 200.5192°**
- B. 200.6982°
- C. 200.6667°
- D. 200.5417°
- E. None of the above

1st method

$$\frac{9''}{3600''} = 0.0025$$

$$\frac{31'}{60'} = +0.5167$$

$$200.5192^\circ$$

2nd method

$$\frac{9}{60} + \frac{31}{60} = 0.5192$$

$$200.5192^\circ$$

3. Find the area of the sector of a circle determined by the central angle θ that subtends an arc of length 13.8 cm on a circle with radius 5.2 cm. Round to the nearest tenth.

- A. 16.9 cm^2
- B. 11.7 cm^2
- C. 35.9 cm^2**
- D. 27.8 cm^2
- E. None of the above

$$A = \frac{1}{2} r^2 \theta \qquad S = r\theta$$

$$13.8 = 5.2\theta$$

$$\theta = \frac{13.8}{5.2} = 2.6538$$

$$A = \frac{1}{2} (5.2^2) (2.65) = 35.88$$

Covers Lessons 1 to 9, Sections 6.1, 6.2, 6.3, and 6.4

4. Let $P(t) = \left(\frac{7}{25}, \frac{-24}{25}\right)$ be the point on the unit circle that corresponds to t .

Find $P(-t + \pi)$.

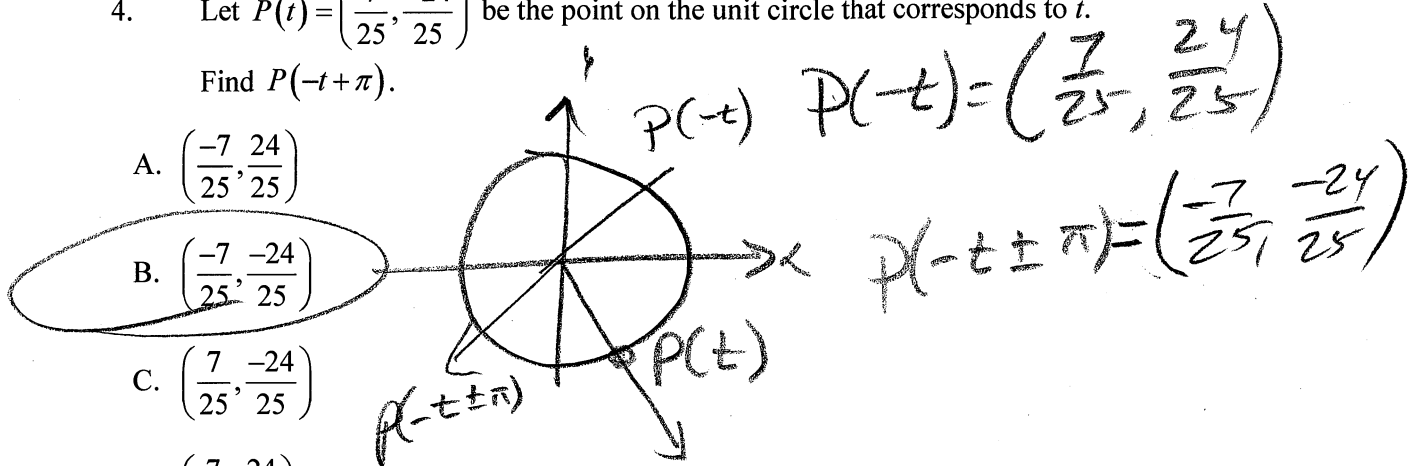
A. $\left(\frac{-7}{25}, \frac{24}{25}\right)$

B. $\left(\frac{-7}{25}, \frac{-24}{25}\right)$

C. $\left(\frac{7}{25}, \frac{-24}{25}\right)$

D. $\left(\frac{7}{25}, \frac{24}{25}\right)$

E. None of the above.



5. A pendulum in a grandfather clock is 3.5 feet long and swings back and forth along a 7-inch arc. Approximate the angle through which the pendulum passes during one swing. Round your answer to the nearest hundredth of a degree.

Hint: 12 inches = 1 foot

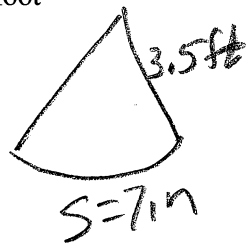
A. 8.47°

B. 7.31°

C. 6.91°

D. 9.55°

E. None of the above



$$s = r\theta$$

$$7 = 42\theta$$

$$\frac{7}{42} = \theta$$

$$\theta = \frac{1}{6}$$

$$\left(\frac{1}{6}\right) \left(\frac{180^\circ}{\pi}\right) = 9.5493^\circ$$

$(3.5)(12) = 42$

9.55°

6. Approximate $\csc(132^\circ)$ to four decimal places.

A. 1.1806

B. -1.9004

C. 1.6824

D. -1.4945

E. None of the above

$$\frac{1}{\sin 132^\circ} = \frac{1}{0.7431} = 1.3456$$

Covers Lessons 1 to 9, Sections 6.1, 6.2, 6.3, and 6.4

7. Find $\cos \theta$.

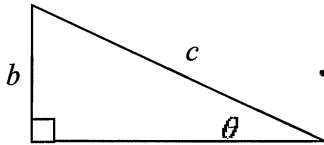
A. $\cos \theta = \frac{\sqrt{c^2 - b^2}}{b}$

B. $\cos \theta = \frac{b}{c}$

C. $\cos \theta = \frac{\sqrt{c^2 - b^2}}{c}$

D. $\cos \theta = \frac{b}{\sqrt{c^2 - b^2}}$

E. $\cos \theta = \frac{c}{b}$



$\sin \theta = \frac{b}{c}$

* $\cos \theta = \frac{\sqrt{c^2 - b^2}}{c}$

$\tan \theta = \frac{b}{\sqrt{c^2 - b^2}}$

$\sqrt{c^2 - b^2}$
 $c^2 = a^2 + b^2$

$\sqrt{c^2 - b^2} = a$

8. Find the exact value of x .

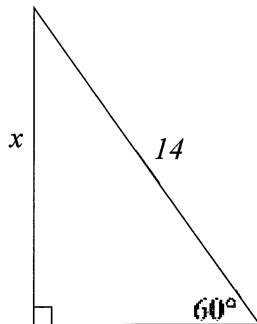
A. $x = 7\sqrt{3}$

B. $x = \frac{28}{\sqrt{3}}$

C. $x = 7$

D. $x = \frac{7}{\sqrt{3}}$

E. None of the above



$\sin 60^\circ = \frac{x}{14}$

$\frac{\sqrt{3}}{2} = \frac{x}{14}$

$2x = 14\sqrt{3}$

$x = 7\sqrt{3}$

9. Find the exact value of $\sec \theta$ if θ is in standard position and the terminal side of θ is in quadrant II and is parallel to the line $8x + y = 4$.

A. $\frac{8}{\sqrt{65}}$

B. $-\frac{1}{2}$

C. 8

D. $-\sqrt{65}$

E. None of the above.

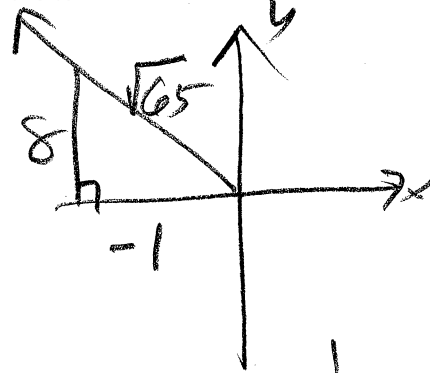
$y = -8x + 4$

$m = -\frac{8}{1}$

$\therefore \tan \theta = -\frac{8}{1}$

$\tan \theta = \frac{y}{x} = -\frac{8}{1}$

$8^2 = 1^2 + b^2$
 $b = \sqrt{65}$



$\cos \theta = -\frac{1}{\sqrt{65}}$

$\sec \theta = -\sqrt{65}$

Covers Lessons 1 to 9, Sections 6.1, 6.2, 6.3, and 6.4

10. $\tan^2 \theta (\cot^2 \theta + 1)$ is equivalent to which of the following?

A. $\csc^2 \theta$

B. $\sec^2 \theta$

C. $\sin^2 \theta$

D. $\cot^2 \theta$

E. $\cos^2 \theta$

$$1 + \tan^2 \theta = \sec^2 \theta$$

11. Complete the statement. As $x \rightarrow \frac{-3\pi^+}{2}$, $\tan(x) \rightarrow$ _____.

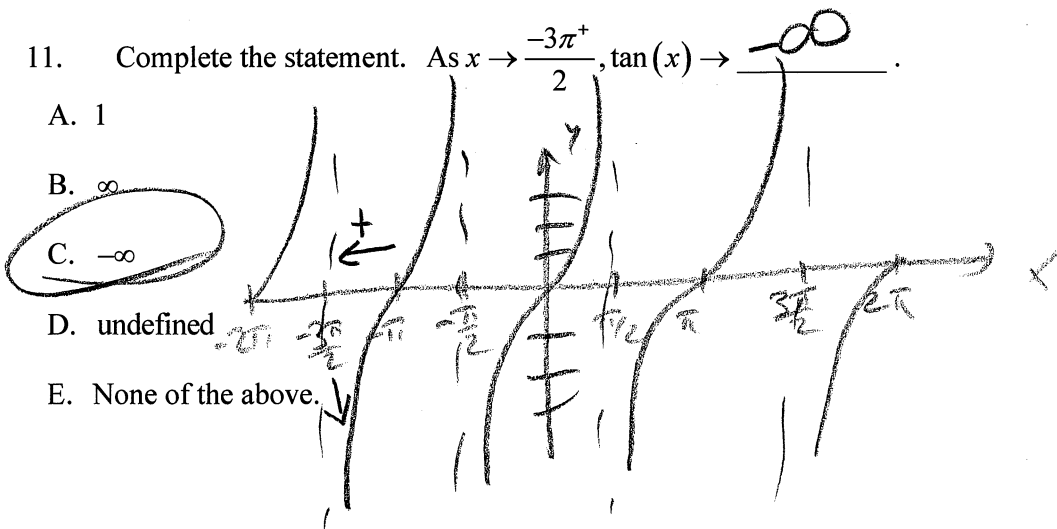
A. 1

B. ∞

C. $-\infty$

D. undefined

E. None of the above.



12. Find the reference angle, θ_R , for $\theta = 845^\circ$

A. 35°

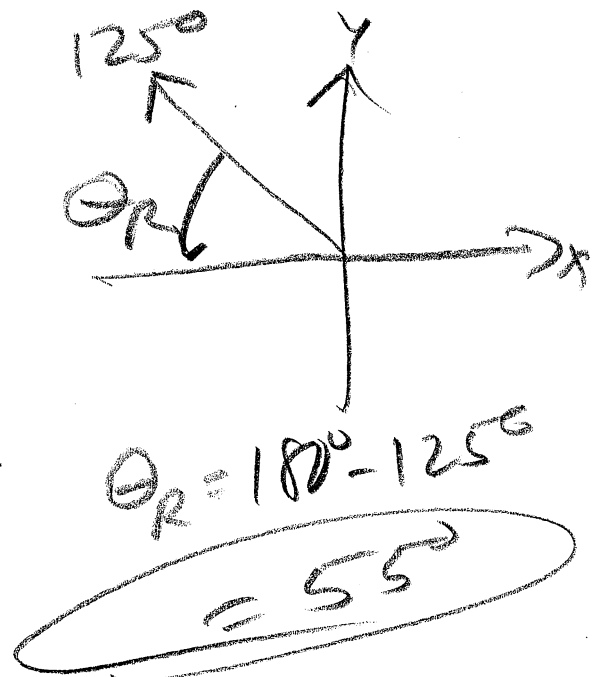
B. 55°

C. 75°

D. 125°

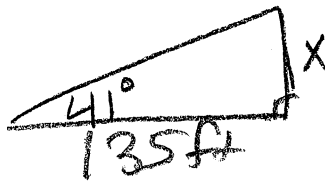
E. None of the above

$$\begin{array}{r} 845^\circ \\ -360^\circ \\ \hline 485^\circ \\ -360^\circ \\ \hline 125^\circ \end{array}$$



13. One day, a long time ago, a farmer traveled 40 miles to see the county's prettiest barn. Being mathematically minded, he found the height of the barn by picking a point, along the level ground, which was 135 feet from a point on the ground directly below the top of the barn and then measured the angle between the ground and the line of sight to the top of the barn to be 41° . To the nearest foot, what is the height of the barn?

- A. 97 feet
- B. 102 feet
- C. 89 feet
- D. 117 feet
- E. None of the above



$$\tan 41^\circ = \frac{x}{135}$$

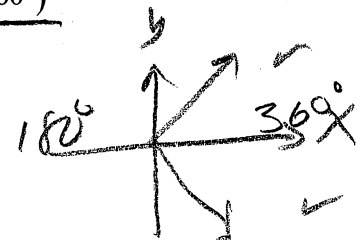
$$135 \tan 41^\circ = x$$

$$x = 117.3537$$

14. Approximate, to the nearest 0.1° , all angles θ in the interval $[0^\circ, 360^\circ)$ that satisfy the equation $\sec(\theta) = 5.2148$.

- A. $82.6^\circ, 281.1^\circ$
- B. $82.6^\circ, 258.9^\circ$
- C. $78.9^\circ, 281.1^\circ$
- D. $78.9^\circ, 258.9^\circ$
- E. None of the above.

$$\frac{1}{\sec \theta} = \frac{1}{5.2148}$$



$$\cos \theta = 0.1918$$

$$\theta = \cos^{-1}(0.1918)$$

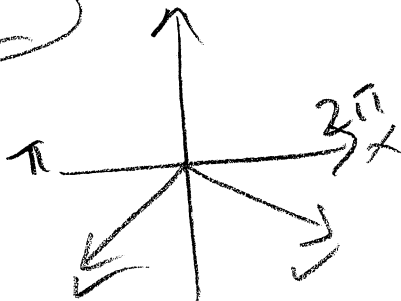
$$\theta_2 = 360^\circ - 79.9^\circ$$

$$\theta_1 = 79.9^\circ$$

$$\theta_2 = 281.1^\circ$$

15. Approximate, to the nearest 0.01 radians, all angles θ in the interval $[0, 2\pi)$ that satisfy the equation $\sin(\theta) = -0.5792$.

- A. 3.76, 5.67
- B. 2.19, 5.33
- C. 2.19, 5.67
- D. 3.76, 5.33
- E. None of the above.



$$\theta = \sin^{-1}(-0.5792)$$

$$\theta = -0.6175$$

$$\theta_2 = 0.6175$$

$$\theta_1 = \pi + 0.6175 = 3.76$$

$$\theta_2 = 2\pi - 0.6175 = 5.67$$

Exam 1 Answers

1.	-300°	B
2.	200.5192°	A
3.	35.9 cm^2	C
4.	$\left(\frac{-7}{25}, \frac{-24}{25}\right)$	B
5.	9.55°	D
6.	1.3456	E
7.	$\cos \theta = \frac{\sqrt{c^2 - b^2}}{c}$	C
8.	$x = 7\sqrt{3}$	A
9.	$-\sqrt{65}$	D
10.	$\sec^2 \theta$	B
11.	$-\infty$	C
12.	55°	B
13.	117 feet	D
14.	$78.9^\circ, 281.1^\circ$	C
15.	3.76, 5.67	A