

# MA 15400

## Fall 2014

# Exam 1

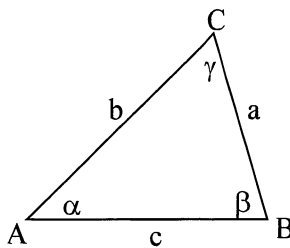
Solutions

PYTHAGOREAN IDENTITIES:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$



Covers Lessons 1-11, Sections 6.1, 6.2, 6.3, 6.4, and 6.5

1. Find the angle that is complementary to  $48^\circ 57' 9''$

$$\begin{array}{r} 90^\circ \\ - 48^\circ 57' 9'' \\ \hline 41^\circ 2' 51'' \end{array}$$

- A.  $42^\circ 3' 51''$
- B.  $131^\circ 2' 51''$
- C.  $41^\circ 2' 51''$
- D.  $132^\circ 3' 51''$
- E. None of the above

2. Express  $\theta = 3.5$  in terms of degrees, minutes, and seconds, to the nearest second.

$$\left(\frac{3.5}{1}\right)\left(\frac{180^\circ}{\pi}\right) = 200.5352^\circ$$

$$\begin{array}{r} 0.5352^\circ \\ \times 60'' \\ \hline 32.1137' \end{array}$$

$$\begin{array}{r} 0.1137' \\ \times 60'' \\ \hline 6.8219'' \end{array}$$

$$200^\circ 32' 7''$$

- A.  $200^\circ 32' 7''$
- B.  $151^\circ 15' 22''$
- C.  $200^\circ 53' 52''$
- D.  $151^\circ 42' 33''$
- E. None of the above

3. Find the measure of the central angle  $\theta$ , to nearest  $0.1^\circ$ , subtended by the arc of length  $s = 3.5$  feet on a circle of radius  $r = 18$  inches. (12 inches = 1 foot)

$$s = r\theta$$

$$\begin{array}{r} 3.5 \text{ ft} \\ \times 12 \text{ in/ft} \\ \hline 42 \text{ in} \end{array}$$

$$42 = 18\theta$$

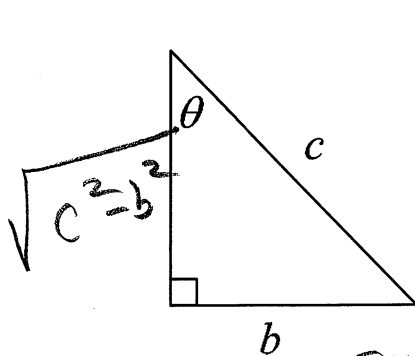
$$\theta = \frac{42}{18} = 2.3 \text{ RADIANS}$$

$$\left(\frac{2.3}{1}\right)\left(\frac{180^\circ}{\pi}\right) = 133.6902^\circ$$

- A.  $115.9^\circ$
- B.  $127.8^\circ$
- C.  $151.5^\circ$
- D.  $133.7^\circ$
- E. None of the above

Covers Lessons 1-11, Sections 6.1, 6.2, 6.3, 6.4, and 6.5

4. Which one of the following statements is true for the given triangle?



$$c^2 = b^2 + a^2$$

$$c^2 - b^2 = a^2$$

$$\sqrt{c^2 - b^2} = a$$

$$\sin \theta = \frac{b}{c}$$

$$\cos \theta = \frac{\sqrt{c^2 - b^2}}{c}$$

$$\tan \theta = \frac{b}{\sqrt{c^2 - b^2}}$$

$$\csc \theta = \frac{c}{b}$$

$$\sec \theta = \frac{c}{\sqrt{c^2 - b^2}}$$

$$\cot \theta = \frac{\sqrt{c^2 - b^2}}{b}$$

~~A.  $\cot \theta = \frac{c}{b}$~~

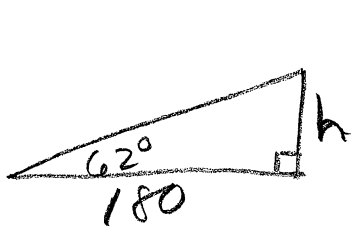
**B.  $\sec \theta = \frac{c}{\sqrt{c^2 - b^2}}$**

~~C.  $\tan \theta = \frac{\sqrt{c^2 - b^2}}{b}$~~

~~D.  $\cos \theta = \frac{b}{\sqrt{c^2 - b^2}}$~~

~~E.  $\csc \theta = \frac{b}{c}$~~

5. A forester, 180 feet from the base of a redwood tree, observes that the angle between the ground and the top of the tree is  $62^\circ$ . Find the height of the tree to the nearest whole foot.



$$\tan 62^\circ = \frac{h}{180}$$

$$180 \tan 62^\circ = h$$

$$h = 338.5308$$

A. 85 feet

B. 159 feet

C. 204 feet

**D. 339 feet**

E. None of the above

6. Which of the following is equivalent to  $(\cot \theta + \csc \theta)(\tan \theta - \sin \theta)$ ?

$$\left( \frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta} \right) \left( \frac{\sin \theta}{\cos \theta} - \frac{\sin \theta}{1} \right)$$

$$1 - \cos \theta + \frac{1}{\cos \theta} - 1$$

$$- \cos \theta + \sec \theta$$

$$\sec \theta - \cos \theta$$

**A.  $\sec \theta - \cos \theta$**

B.  $1 + \tan^2 \theta$

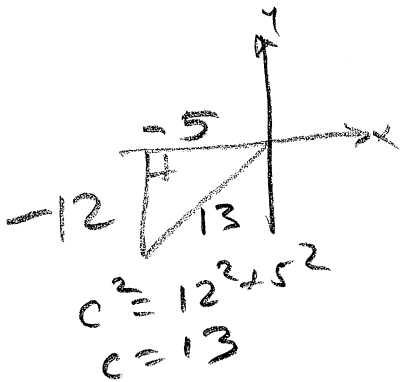
C.  $\csc \theta - \sin \theta$

D.  $\csc \theta \sec \theta$

E.  $1 + \cot^2 \theta$

Covers Lessons 1-11, Sections 6.1, 6.2, 6.3, 6.4, and 6.5

7. Find the exact value of  $\sin \theta$  if  $\theta$  is in standard position and the terminal side of  $\theta$  is in quadrant III and parallel to the line  $12x - 5y = 15$



$c^2 = 12^2 + 5^2$   
 $c = 13$

$\sin \theta = \frac{y}{r} = \frac{-12}{13}$

Find Slope

Slope =  $\tan \theta$

$-5y = -12x + 15$

$y = \frac{12}{5}x - 3$

$m = \frac{12}{5}, \tan \theta = \frac{12}{5}$

$\tan \theta = \frac{y}{x} = \frac{-12}{-5}$

A.  $\sin \theta = \frac{5}{13}$

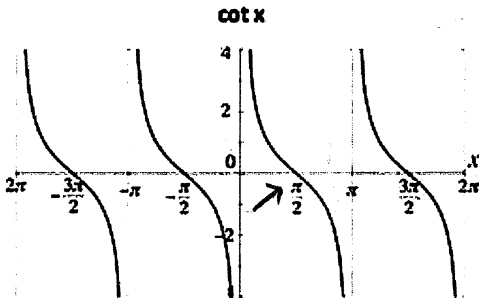
B.  $\sin \theta = \frac{-12}{13}$

C.  $\sin \theta = \frac{-5}{13}$

D.  $\sin \theta = \frac{12}{13}$

E. None of the above

8. Use the graph to complete the statement: As  $x \rightarrow \frac{\pi}{2}$ ,  $\cot(x) \rightarrow$  \_\_\_\_\_



A.  $-\infty$

B. 1

C. 0

D.  $\infty$

E. None of the above

9. In March in Tucson, Arizona, the temperature in degrees Fahrenheit could be described by the equation  $T(t) = -11 \cos\left(\frac{\pi}{12}t\right) + 57$ , where  $t$  is in hours and  $t = 0$  corresponds to 6 A.M.

What is the temperature at 3 P.M.? Angles are in radians and round to a whole number.

Time	$t$
6 Am	0
9 Am	3
12 Pm	6
3pm	9

$t = 9$   
 $T(9) = -11 \cos\left(\frac{\pi}{12} \cdot 9\right) + 57$   
 $= 64.7782$

A.  $49^\circ$

B.  $57^\circ$

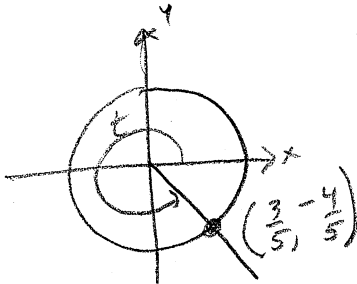
C.  $68^\circ$

D.  $54^\circ$

E.  $65^\circ$

Covers Lessons 1-11, Sections 6.1, 6.2, 6.3, 6.4, and 6.5

10. A point  $P\left(\frac{3}{5}, \frac{-4}{5}\right)$  is the point of intersection between the terminal side of angle  $t$  and the Unit circle. Find the exact value of  $\cot(t)$



$\sin t = -4/5$   
 $\cos t = 3/5$

$\cot t = \frac{\cos t}{\sin t} = \frac{3/5}{-4/5} = -\frac{3}{4}$

A.  $\cot(t) = \frac{3}{5}$

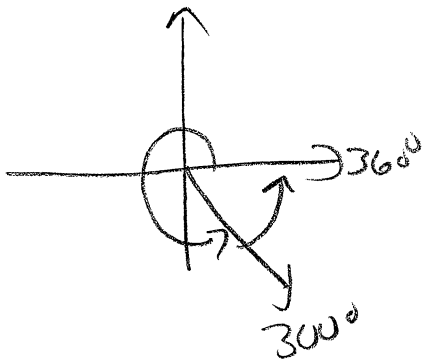
B.  $\cot(t) = \frac{-4}{5}$

C.  $\cot(t) = \frac{4}{3}$

D.  $\cot(t) = \frac{-3}{4}$

E. None of the above

11. Find the reference angle  $\theta_R$  if  $\theta = 300^\circ$



$\theta_R = 360^\circ - 300^\circ = 60^\circ$

A.  $\theta_R = 60^\circ$

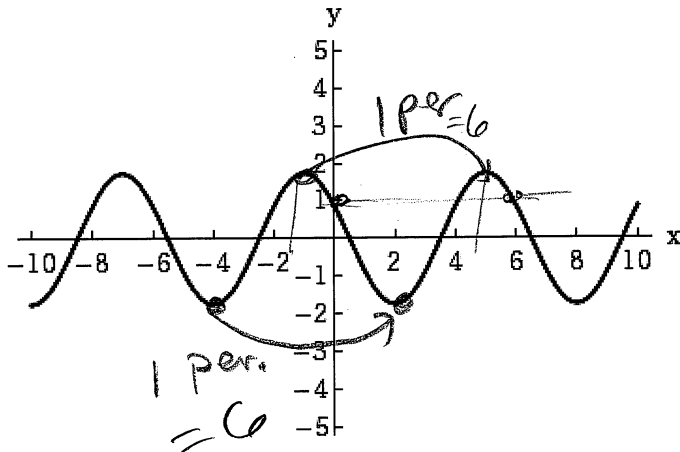
B.  $\theta_R = 30^\circ$

C.  $\theta_R = 120^\circ$

D.  $\theta_R = 45^\circ$

E. None of the above

12. Find the Period of the given graph.



A. Period = 8

B. Period = 6

C. Period = 4

D. Period = 2

E. Period =  $\infty$

Covers Lessons 1-11, Sections 6.1, 6.2, 6.3, 6.4, and 6.5

13. Write the equation in the form  $y = a \sin(bx + c)$  for  $a > 0$ ,  $b > 0$ , and the least positive real number  $c$ .

$\text{Amp} = 1$   
 $a = 1$   
 $\text{Per} = 6\pi$   
 $6\pi = \frac{2\pi}{b}$   
 $b = \frac{2\pi}{6\pi} = \frac{1}{3}$   
 $\text{P.S.} = -3\pi$   
 $-3\pi = \frac{-c}{b}$   
 $-3\pi = \frac{-c}{\frac{1}{3}}$   
 $c = \pi$   
 $y = \sin\left(\frac{1}{3}x + \pi\right)$

A.  $y = \sin\left(\frac{2}{3}x + \pi\right)$   
 B.  $y = \sin\left(\frac{1}{3}x + 2\pi\right)$   
 C.  $y = \sin\left(\frac{2}{3}x + 2\pi\right)$   
 D.  $y = \sin\left(\frac{1}{3}x + \pi\right)$   
 E.  $y = \sin(6x + 3\pi)$

14. Approximate, to the nearest  $0.1^\circ$ , all angles  $\theta$  in the interval  $[0^\circ, 360^\circ)$  that satisfy equation  $\sec \theta = 2.3456$ .

$\frac{1}{\sec \theta} = \frac{1}{2.3456}$   
 $\cos \theta = 0.4263$   
 $\theta_1 = \cos^{-1}(0.4263)$   
 $\theta_1 = 64.7651^\circ$   
 $\theta_2 = 360^\circ - 64.8^\circ$   
 $\theta_2 = 295.2^\circ$

A.  $\theta = 64.8^\circ, 295.2^\circ$   
 B.  $\theta = 115.2^\circ, 244.8^\circ$   
 C.  $\theta = 64.8^\circ, 244.8^\circ$   
 D.  $\theta = 115.2^\circ, 295.2^\circ$   
 E. None of the above

15. Approximate, to the nearest 0.0001 radians, all angles  $\theta$  in the interval  $[0, 2\pi)$  that satisfy equation  $\sin \theta = -0.8765$

$\theta = \sin^{-1}(-0.8765)$   
 $\theta = 1.0685$   
 $\theta_1 = \pi + 1.0685 = 4.2101$   
 $\theta_2 = 2\pi - 1.0685 = 5.2146$

A.  $\theta = 1.0685, 5.2146$   
 B.  $\theta = 2.0730, 4.2101$   
 C.  $\theta = 4.2101, 5.2146$   
 D.  $\theta = 1.0685, 2.0730$   
 E. None of the above

Question	Answer	Letter
1.	$41^{\circ}2'51''$	C
2.	$200^{\circ}32'7''$	A
3.	$133.7^{\circ}$	D
4.	$\sec \theta = \frac{c}{\sqrt{c^2 - b^2}}$	B
5.	339 feet	D
6.	$\sec \theta - \cos \theta$	A
7.	$\sin \theta = \frac{-12}{13}$	B
8.	0	C
9.	$65^{\circ}$	E
10.	$\cot(t) = \frac{-3}{4}$	D
11.	$\theta_R = 60^{\circ}$	A
12.	Period = 6	B
13.	$y = \sin\left(\frac{1}{3}x + \pi\right)$	D
14.	$\theta = 64.8^{\circ}, 295.2^{\circ}$	A
15.	$\theta = 4.2101, 5.2146$	C