MA 15400

Fall 2014

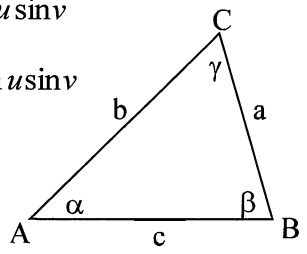
Exam 2

Solutions

$\sin(u+v)=\sin u\cos v+\cos u\sin v$

$\cos(u+v)=\cos u\cos v-\sin u\sin v$

$\tan(u+v)=\dfrac{\tan u+\tan v}{1-\tan u\tan v}$



$\sin(u-v)=\sin u\cos v-\cos u\sin v$

$\cos(u-v)=\cos u\cos v+\sin u\sin v$

$\tan(u-v)=\dfrac{\tan u-\tan v}{1+\tan u\tan v}$

$\sin(2u)=2\sin u\cos u$

$\cos(2u)=\cos^2 u-\sin^2 u$

$\tan(2u)=\dfrac{2\tan u}{1-\tan^2 u}$

$\sin^2\theta+\cos^2\theta=1$

$1+\tan^2\theta=\sec^2\theta$

$1+\cot^2\theta=\csc^2\theta$

MA 15400 Exam 2 Fall 2014

Lessons 12-20, Covers Sections 6.7, 7.2, 7.3, and 7.4

1. Given $\triangle ABC$ with $\gamma = 90°, \alpha = 30°$, and $b = 15$, find the exact value of side a.

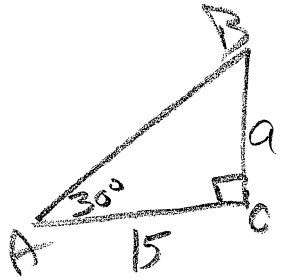
A. $15\sqrt{3}$

B. $10\sqrt{3}$

C. $\frac{5}{\sqrt{3}}$

D. $\frac{15}{2}$

E. $5\sqrt{3}$



$\tan 30° = \frac{a}{15}$

$15(\tan 30°) = a$

$15\left(\frac{1}{\sqrt{3}}\right) = a$

$a = \frac{15}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{3} = 5\sqrt{3}$

2. Given $\triangle ABC$ with $\gamma = 90°$, angle $\alpha = 21°$, and side $b = 8.6$, approximate the perimeter of the triangle to the nearest tenth.

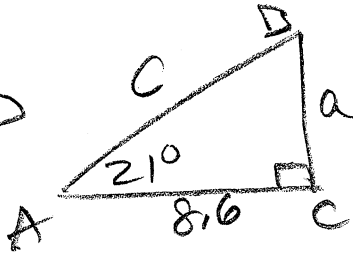
A. 17.4

B. 21.1

C. 19.9

D. 18.1

E. 23.6



$\tan 21° = \frac{a}{8.6}$

$8.6 \tan 21° = a$

$a = 3.3012$

$c^2 = 8.6^2 + 3.3^2$

$c^2 = 84.8501$

$c = 9.2118$

$P = 3.3 + 9.2 + 8.6 = 21.1$

3. Given the indicated parts of $\triangle ABC$ with $\gamma = 90°$, express the third part in terms of the first two.

a, β; c

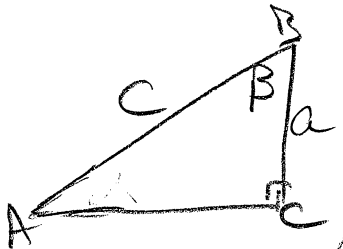
A. $c = a \tan \beta$

B. $c = a \csc \beta$

C. $c = a \sec \beta$

D. $c = a \cos \beta$

E. $c = a \sin \beta$



$\cos B = \frac{a}{c}$

$c = \frac{a}{\cos B} = a \sec B$

4. From a distance of 1 mile on level ground, a certain tower has an angle of elevation of 8°. Determine its height to the nearest foot. (1 mile = 5280 feet)

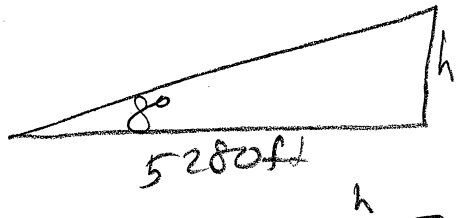
A. 742 *feet*

B. 836 *feet*

C. 734 *feet*

D. 826 *feet*

E. None of the above



$\tan 8° = \frac{h}{5280}$

$h = 5280(\tan 8°)$

$h = 742.0556$

5. A ladder, 30 feet long, leans against the side of a building, and the angle between the ladder and the building is 16°.

If the distance from the bottom of the ladder to the building is increase by 3.0 feet, approximate the angle the ladder now makes with the building to the nearest 0.1°.

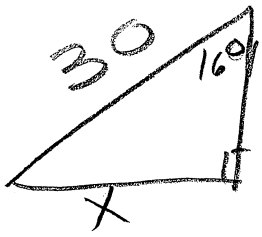
A. 10.1°

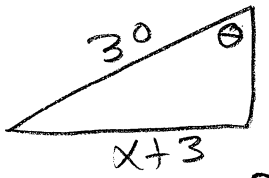
B. 10.5°

C. 22.1°

D. 21.7°

E. None of the above





$\sin 16° = \frac{X}{30}$

$X = 30 \sin 16°$

$X = 8.2691$

$X + 3 = 11.2681$

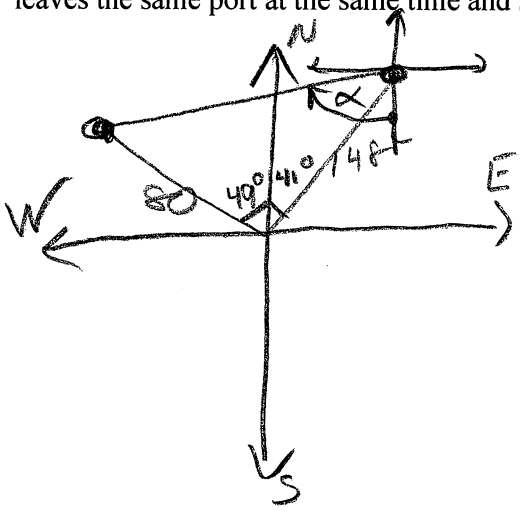
$\sin\theta = \frac{11.2681}{30}$

$\theta = \sin^{-1}\left(\frac{11.2681}{30}\right)$

$\theta = 22.0637°$

Questions 6 and 7. The following is a love story of two ships that do not pass in the night.

A ship leave port at 2:00 pm and sails in the direction N 41° E at a rate of 37 mph. Another ship leaves the same port at the same time and sails in the direction N 49° W at a rate of 20 mph.



$D = rt \quad t = 4 \text{ hrs}$

$D_1 = 37(4) = 148 \text{ mi}$

$D_2 = 20(4) = 80 \text{ mi}$

6. To the nearest whole mile, approximately how far apart are the ships at 6:00 pm?

A. 136 *miles*

B. 126 *miles*

C. 175 *miles*

D. 168 *miles*

E. None of the above

$x^2 = 148^2 + 80^2$

$x = 168.2379 \text{ mi}$

7. What is the bearing, to the nearest degree, from the first ship to the second?

A. *S*78°*W*

B. *S*66°*W*

C. *S*69°*W*

D. *S*13°*W*

E. None of the above

$S\ 41° + \alpha\ W$

$\tan\alpha = \frac{80}{148}$

$\alpha = \tan^{-1}\left(\frac{80}{148}\right)$

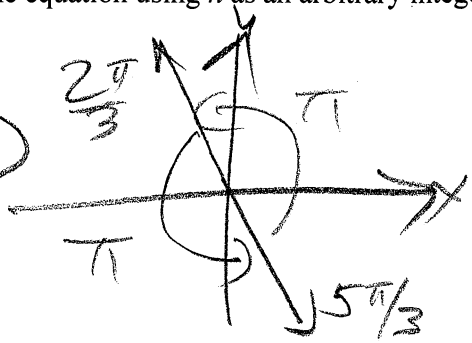
$\alpha = 28.3930°$

$+ 41.0°$

$69.3930°$

8. Find all solutions of the equation using n as an arbitrary integer.

$$\tan\theta = -\sqrt{3}$$



A. $\theta = \frac{2\pi}{3} + \pi n$

B. $\theta = \frac{\pi}{3} + 2\pi n$

C. $\theta = \frac{2\pi}{3} + 2\pi n$

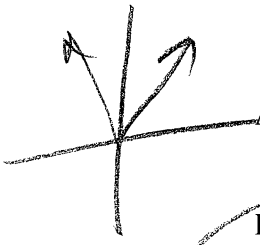
D. $\theta = \frac{\pi}{3} + \pi n$

$\theta = \frac{2\pi}{3} + \pi n$

E. None of the above

9. Find all solutions of the equation using n as an arbitrary integer.

$$\sin(2x) = \frac{\sqrt{3}}{2}$$



$2x = \frac{\pi}{3} + 2\pi n$ $\quad 2x = \frac{2\pi}{3} + 2\pi n$

$x = \frac{\pi}{6} + \pi n$ $\quad x = \frac{\pi}{2} + \pi n$

A. $\theta = \frac{\pi}{8} + \pi n, \frac{3\pi}{8} + \pi n$

B. $\theta = \frac{\pi}{6} + \pi n, \frac{\pi}{3} + \pi n$

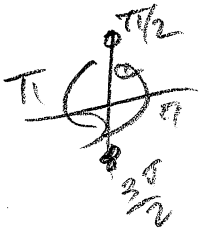
C. $\theta = \frac{2\pi}{3} + \pi n, \frac{5\pi}{6} + \pi n$

D. $\theta = \frac{\pi}{6} + \pi n, \frac{5\pi}{6} + \pi n$

E. None of the above

10. Find all solutions of the equation in the interval $[0,2\pi)$

$$\cos\left(2x-\frac{\pi}{4}\right)=0$$



A. $\theta=\frac{5\pi}{24},\frac{17\pi}{24},\frac{29\pi}{24},\frac{41\pi}{24}$

B. $\theta=\frac{\pi}{8},\frac{5\pi}{8},\frac{9\pi}{8},\frac{13\pi}{8}$

C. $\theta=\frac{7\pi}{24},\frac{19\pi}{24},\frac{31\pi}{24},\frac{43\pi}{24}$

D. $\theta=\frac{3\pi}{8},\frac{7\pi}{8},\frac{11\pi}{8},\frac{15\pi}{8}$

E. None of the above

$2x-\frac{\pi}{4}=\frac{\pi}{2}+\pi n$

$2x=\frac{3\pi}{4}+\pi n$

$x=\frac{3\pi}{8}+\frac{\pi}{2}n$

$x=\frac{3\pi}{8},\frac{4\pi}{8}n$

$\frac{\pi}{4}+\frac{\pi}{2}=\frac{3\pi}{4}$

$x=\frac{3\pi}{8},\frac{7\pi}{8},\frac{11\pi}{8},\frac{15\pi}{8}$

n	x
0	$3\pi/8$
1	$7\pi/8$
2	$11\pi/8$
3	$15\pi/8$
4	TOO BIG
-1	TOO SMALL

11. Find all solutions of the equation in the interval $[0,2\pi)$

$$2\sin^2\theta-3\sin\theta+1=0$$

A. $\theta=\frac{\pi}{6},\frac{5\pi}{6},\frac{3\pi}{2}$

B. $\theta=\frac{7\pi}{6},\frac{11\pi}{6},\frac{\pi}{2}$

C. $\theta=\frac{\pi}{6},\frac{5\pi}{6},\frac{\pi}{2}$

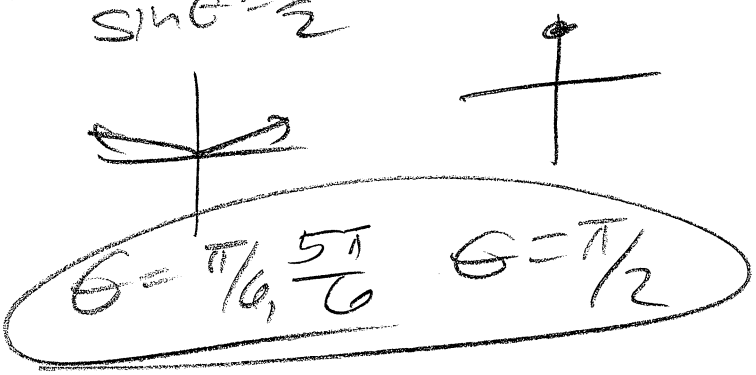
D. $\theta=\frac{7\pi}{6},\frac{11\pi}{6},\frac{3\pi}{2}$

E. None of the above

$(2\sin\theta-1)(\sin\theta-1)=0$

$2\sin\theta-1=0$ $\quad\sin\theta-1=0$

$\sin\theta=\frac{1}{2}$ $\quad\sin\theta=1$



$\theta=\pi/6,\frac{5\pi}{6}$ $\quad\theta=\pi/2$

12. Express as a trigonometric function of one angle.

$$\sin(35°)\cos(15°)-\cos(35°)\sin(15°)$$

A. $\cos(50°)$

B. $\sin(20°)$

C. $\cos(20°)$

D. $\sin(50°)$

E. None of the above

$\sin(u-v)$

$\sin(35°-15°)$

$\sin(20°)$

13. Find all solutions of the equation in the interval $[0,2\pi)$.

$$\sin t-\sin 2t=0$$

A. $t=\frac{\pi}{2},\frac{3\pi}{2},\frac{2\pi}{3},\frac{4\pi}{3}$

B. $t=0,\pi,\frac{\pi}{3},\frac{5\pi}{3}$

C. $t=\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{3},\frac{5\pi}{3}$

D. $t=0,\pi,\frac{2\pi}{3},\frac{4\pi}{3}$

E. None of the above

$\sin t - 2\sin t\cos t = 0$

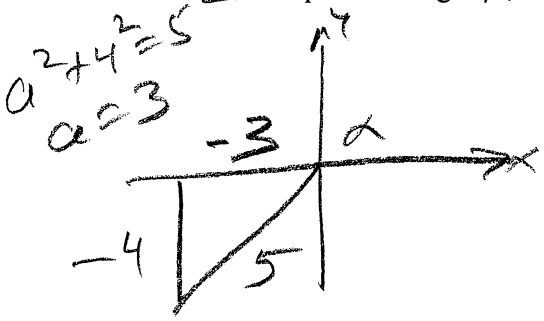
$\sin t(1-2\cos t)=0$

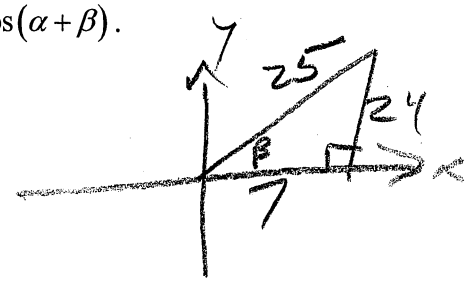
$\sin t=0 \quad 1-2\cos t=0$

$t=0,\pi \quad \cos t=1/2$

$t=\pi/3,\frac{5\pi}{3}$

14. If $\sin\alpha = \frac{-4}{5}$ and $\cos\beta = \frac{7}{25}$, for a **third**-quadrant angle α and a **first**-quadrant angle β, find $\cos(\alpha+\beta)$.





$25^2 = a^2 + 7^2$
$a = 24$

A. $\frac{-4}{5}$

B. $\frac{-117}{125}$

C. $\frac{44}{125}$

D. $\frac{3}{5}$

E. None of the above

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{-3}{5}\right)\left(\frac{7}{25}\right) - \left(\frac{-4}{5}\right)\left(\frac{24}{25}\right)$

$= \frac{-21}{125} - \frac{-96}{125} = \frac{-21+96}{125}$

$= \frac{75}{125} = \frac{3}{5}$

15. Find the exact value of $\cos 2\theta$ if $\tan\theta = \frac{-5}{6}$ and $270° < \theta < 360°$

A. $\frac{-11}{61}$

B. $\frac{-60}{61}$

C. $\frac{5}{61}$

D. $\frac{60}{61}$

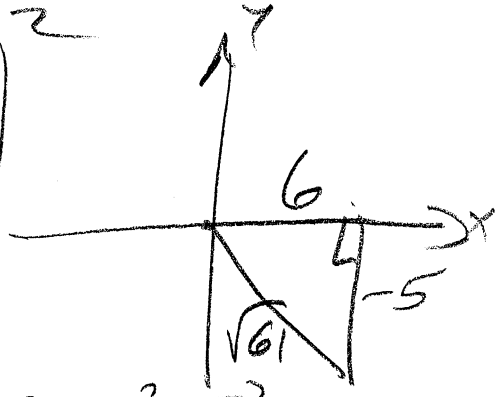
E. None of the above

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{6}{\sqrt{61}}\right)^2 - \left(\frac{-5}{\sqrt{61}}\right)^2$

$= \frac{36}{61} - \frac{25}{61}$

$= \frac{11}{61}$



$c^2 = 6^2 + 5^2$
$c = \sqrt{61}$

Lessons 12-20, Covers Sections 6.7, 7.2, 7.3, and 7.4

Question	Answers	Letters
1.	$5\sqrt{3}$	E
2.	21.1	B
3.	$c = a\sec\beta$	C
4.	742 *feet*	A
5.	22.1°	C
6.	168 *miles*	D
7.	$S69°W$	C
8.	$\theta = \frac{2\pi}{3} + \pi n$	A
9.	$\theta = \frac{\pi}{6} + \pi n, \frac{\pi}{3} + \pi n$	B
10.	$\theta = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$	D
11.	$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$	C
12.	$\sin(20°)$	B
13.	$t = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$	B
14.	$\frac{3}{5}$	D
15.	$\frac{11}{61}$	E