MA 15400

Spring 2013

Exam 2

$$\sin(u+v)=\sin u\cos v+\cos u\sin v \qquad \sin(u-v)=\sin u\cos v-\cos u\sin v$$

$$\cos(u+v)=\cos u\cos v-\sin u\sin v \qquad \cos(u-v)=\cos u\cos v+\sin u\sin v$$

$$\tan(u+v)=\frac{\tan u+\tan v}{1-\tan u\tan v} \qquad \tan(u-v)=\frac{\tan u-\tan v}{1+\tan u\tan v}$$

$$\sin(2u)=2\sin u\cos u \qquad \cos(2u)=\cos^2 u-\sin^2 u \qquad \tan(2u)=\frac{2\tan u}{1-\tan^2 u}$$

$$\sin^2\theta+\cos^2\theta=1$$

$$1+\tan^2\theta=\sec^2\theta \qquad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$1+\cot^2\theta=\csc^2\theta$$

MA 15400 Exam 2 Spring 2013

Lessons 13-23, Section 6.7 starting at question #33, and all of Sections 7.2, 7.3, 7.4 and 7.6

1. Express as a trigonometric function of one angle.

$$\cos(49°)\cos(24°)-\sin(49°)\sin(24°)$$

A. $\cos(73°)$

B. $\sin(25°)$

C. $\sin(73°)$

D. $\cos(25°)$

E. None of the above

$\cos(49°+24°) = \cos 73°$

2. Find all solutions of the equation using n as an arbitrary integer.

$$\sin\left(2x-\frac{\pi}{3}\right)=\frac{1}{2}$$

A. $x=\frac{\pi}{3}+\pi n, \frac{\pi}{2}+\pi n$

B. $x=\frac{\pi}{6}+\pi n, \frac{\pi}{2}+\pi n$

C. $x=\frac{\pi}{4}+\pi n, \frac{5\pi}{12}+\pi n$

D. $x=\frac{\pi}{4}+\pi n, \frac{7\pi}{12}+\pi n$

E. None of the above

$2x-\frac{\pi}{3}=\frac{\pi}{6}+2\pi n$ $\quad 2x-\frac{\pi}{3}=\frac{5\pi}{6}+2\pi n$

$2x=\frac{\pi}{3}+\frac{\pi}{2}+2\pi n$ $\quad 2x=\frac{\pi}{3}+\frac{5\pi}{6}+2\pi n$

$2x=\frac{\pi}{2}+2\pi n$ $\quad 2x=\frac{7\pi}{6}+2\pi n$

$x=\frac{\pi}{4}+\pi n$ $\quad x=\frac{7\pi}{12}+\pi n$

3. Find the solutions of the equation that are in the interval $[0, 2\pi)$.

$$2\sin^2 u=-1+3\sin u$$

A. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{\pi}{2}$

B. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$

C. $\frac{4\pi}{3}, \frac{5\pi}{3}, \frac{3\pi}{2}$

D. $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$

E. None of the above

$2\sin^2 u-3\sin u+1=0$

$(2\sin u-1)(\sin u-1)=0$

$2\sin u-1=0$ $\quad \sin u-1=0$

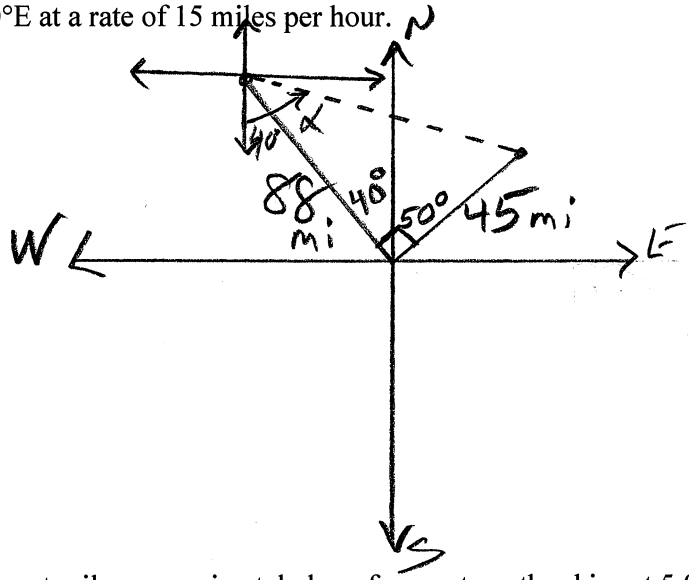
$\sin u=\frac{1}{2}$ $\quad \sin u=1$

$u=\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$

MA 15400 Exam 2 Spring 2013

Lessons 13-23, Section 6.7 starting at question #33, and all of Sections 7.2, 7.3, 7.4 and 7.6

4 and 5: A ship leaves port at 1:00 pm and sails in the direction N40°W at a rate of 22 miles per hour. A second ship leaves the same port at 2:00 pm. and sails in the direction N50°E at a rate of 15 miles per hour.



$D = rt$

$D_1 = 22(4) = 88$

$D_2 = 15(3) = 45$

$40° + 50° = 90°$

4. To the nearest mile, approximately how far apart are the ships at 5:00 pm?

A. 80 miles
B. 95 miles
C. 99 miles
D. 107 miles
E. None of the above

$c^2 = 88^2 + 45^2$

$c = 98.8$ miles

5. To the nearest degree, what is the bearing from the first ship to the second at 5:00 pm?

A. S67°E
B. S59°E
C. S27°E
D. S13°E
E. None of the above

Alt. Int Angle $= 40°$

$\tan\alpha = \frac{45}{88}$

$\alpha = \tan\left(\frac{45}{88}\right)$

$\alpha = 27°$

$S\ 40° + \alpha\ E$

$S\ 67°\ E$

Lessons 13-23, Section 6.7 starting at question #33, and all of Sections 7.2, 7.3, 7.4 and 7.6

6. Determine if the identity is correct.

$$\cos\left(\theta+\frac{\pi}{3}\right)=\frac{1}{2}\left(\sqrt{3}\cos\theta-\sin\theta\right)$$

A. Yes, it is correct.

B. No, it is not correct.

$\cos\theta\cos\frac{\pi}{3}-\sin\theta\sin\frac{\pi}{3}$

$\cos\theta\left(\frac{1}{2}\right)-\sin\theta\left(\frac{\sqrt{3}}{2}\right)$

$\frac{1}{2}\left(\cos\theta-\sqrt{3}\sin\theta\right)\neq\frac{1}{2}\left(\sqrt{3}\cos\theta-\sin\theta\right)$

7. Find the solutions of the equation that are in the interval $[0, 2\pi)$.

$$\cos\left(3x-\frac{\pi}{6}\right)=-1$$

A. $\frac{4\pi}{9},\frac{10\pi}{9},\frac{16\pi}{9}$

B. $0,\frac{2\pi}{3},\frac{4\pi}{3}$

C. $\frac{7\pi}{18},\frac{19\pi}{18},\frac{31\pi}{18}$

D. $\frac{3\pi}{8},\frac{9\pi}{8},\frac{15\pi}{8}$

E. None of the above

$3x-\frac{\pi}{6}=\pi+2\pi n$

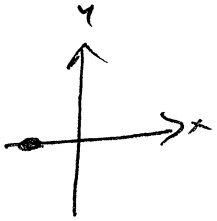
$3x=\frac{\pi}{6}+\frac{6\pi}{6}+2\pi n$

$3x=\frac{7\pi}{6}+2\pi n$

$x=\frac{7\pi}{18}+\frac{2\pi}{3}n$

rewrite with common den.

$x=\frac{7\pi}{18}+\frac{12\pi}{18}n$



n	x
0	$\frac{7\pi}{18}$
1	$\frac{19\pi}{18}$
2	$\frac{31\pi}{18}$
4	TOO BIG
-1	TOO SMALL

8. If $\sin\alpha=\frac{-3}{8}$ and $\cos\beta=\frac{4}{5}$ for a third-quadrant angle α and a first-quadrant angle β, then find the exact value of $\sin(\alpha-\beta)$.

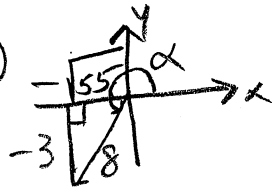
A. $\frac{-12+3\sqrt{55}}{40}$

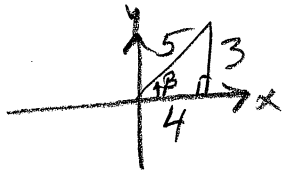
B. $\frac{9-4\sqrt{55}}{40}$

C. $\frac{12-3\sqrt{55}}{40}$

D. $\frac{-9+4\sqrt{55}}{40}$

E. None of the above





$8^2=3^2+b^2$

$b=\pm\sqrt{55}$

$5^2=4^2+a^2$

$a=\pm3$

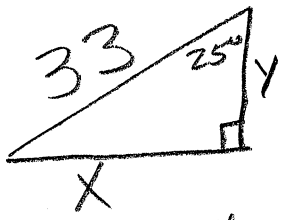
$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

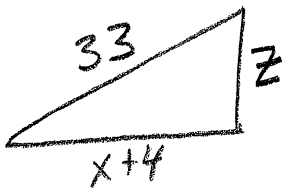
$=\left(\frac{-3}{8}\right)\left(\frac{4}{5}\right)-\left(\frac{-\sqrt{55}}{8}\right)\left(\frac{3}{5}\right)$

$=\frac{-12}{40}-\frac{-3\sqrt{55}}{40}=\frac{-12+3\sqrt{55}}{40}$

9. A ladder 33.0 feet long leans against the side of a building, and the angle between the ladder and the building is 25°. If the distance from the bottom of the ladder to the building is **increased** by 4.0 feet, approximately how far does the top of the ladder move down the building? Give the answer to one decimal place.

A. 1.8 feet
B. 2.2 feet
C. 1.9 feet
D. 2.0 feet
E. None of the above





$\sin 25° = \frac{x}{33}$

$33\sin 25° = x$

$x = 13.946$

$x+4 = 17.946$

$33^2 = x^2 + y^2$

$33^2 = 13.946^2 = y^2$

$y = 29.908$

$33^2 = (x+4)^2 + z^2$

$33^2 = 17.946^2 + z^2$

$z = 27.693$

Answer:

Diff = 29.9 − 27.7

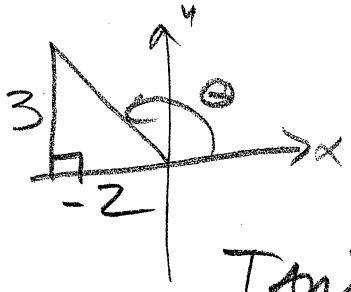
= 2.2

10. Find the exact value of $\tan(2\theta)$ for the given value of θ.

$$\tan\theta = \frac{-3}{2}, \quad 90° < \theta < 180°$$

$\tan\theta = \frac{y}{x} = \frac{-3}{2}$

$x = -2, \; y = 3$



A. $\tan(2\theta) = \frac{20}{21}$

B. $\tan(2\theta) = \frac{-12}{5}$

C. $\tan(2\theta) = \frac{-20}{21}$

D. $\tan(2\theta) = \frac{12}{5}$

E. None of the above

$$\tan 2\theta = \frac{2\left(\frac{-3}{2}\right)}{1-\left(\frac{-3}{2}\right)^2} = \frac{-3}{\frac{4}{4}-\frac{9}{4}}$$

$$= \frac{\frac{-3}{1}}{\frac{-5}{4}} = \frac{-3}{1}\cdot\frac{-4}{5} = \frac{12}{5}$$

MA 15400 Exam 2 Spring 2013

Lessons 13-23, Section 6.7 starting at question #33, and all of Sections 7.2, 7.3, 7.4 and 7.6

11. Find the solutions of the equation that are in the interval $[0, 2\pi)$.

$$\sin t + \sin(2t) = 0$$

A. $t = \frac{\pi}{2}, \frac{3\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{3}$

B. $t = 0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

C. $t = \frac{\pi}{2}, \frac{3\pi}{4}, \frac{2\pi}{3}, \frac{4\pi}{3}$

D. $t = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

E. None of the above

$\sin t + 2\sin t\cos t = 0$

$\sin t(1 + 2\cos t) = 0$

$\sin t = 0$ $\quad 1 + 2\cos t = 0$

$\cos t = -1/2$

$t = 0, \pi$ $\quad t = \frac{2\pi}{3}, \frac{4\pi}{3}$

12. If a projectile is fired from ground level with an initial velocity of v ft/sec and at an angle of θ degrees with the horizontal, the range R of the projectile is given by the following formula. If $v = 87$ ft/sec, approximate the angles, to the nearest whole degree, that result in a range of 151 feet.

$$R = \frac{v^2}{16}\sin\theta\cos\theta$$

$V = 87, R = 151$

A. $\theta = 40°, 50°$

B. $\theta = 21°, 69°$

C. $\theta = 42°, 48°$

D. $\theta = 20°, 70°$

E. None of the above

$151 = \frac{87^2}{16}\sin\theta\cos\theta$

$0.3192 = \sin\theta\cos\theta$

$0.6384 = 2\sin\theta\cos\theta$

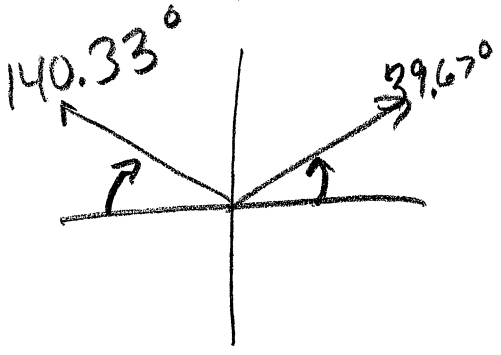
$0.6384 = \sin(2\theta)$

$2\theta = \sin^{-1}(0.6384)$

$2\theta = 39.67°$ $\quad 2\theta = 140.33°$

$\theta = 19.84°$ $\quad \theta = 70.16°$

$180° - 39.67°$



Lessons 13-23, Section 6.7 starting at question #33, and all of Sections 7.2, 7.3, 7.4 and 7.6

13. Find the exact value of the expression whenever it is defined.

$$\cos\left(\sin^{-1}\left(\frac{-1}{2}\right)\right)$$

A. $\frac{-\sqrt{3}}{2}$

B. $\frac{1}{2}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{-1}{2}$

E. None of the above

not on Exam #2 SP '14

14. Write the expression as an algebraic expression in x for $x > 0$.

$$\cos(\tan^{-1} x)$$

A. $\frac{x}{\sqrt{1+x^2}}$

B. $\frac{1}{1+x}$

C. $\frac{x}{1+x}$

D. $\frac{1}{\sqrt{1+x^2}}$

E. None of the above

not on Ex #2 SP '14

15. Approximate the solutions of the equation, to two decimals, that are in the given interval.

$$\sin^2 x - 2\sin x - 2 = 0;\ [0,2\pi)$$

A. 3.96,5.46

B. 2.36,3.18

C. 2.73,3.87

D. 3.57,5.14

E. None of the above

not on Ex. #2 SP '14

Lessons 13-23, Section 6.7 starting at question #33, and all of Sections 7.2, 7.3, 7.4 and 7.6

	Exam 2 Answers	
1.	$\cos(73°)$	A
2.	$x=\frac{\pi}{4}+\pi n, \frac{7\pi}{12}+\pi n$	D
3.	$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$	B
4.	99 miles	C
5.	S67°E	A
6.	No, it is not correct.	B
7.	$\frac{7\pi}{18}, \frac{19\pi}{18}, \frac{31\pi}{18}$	C
8.	$\frac{-12+3\sqrt{55}}{40}$	A
9.	2.2 feet	B
10.	$\tan(2\theta)=\frac{12}{5}$	D
11.	$t=0,\pi,\frac{2\pi}{3},\frac{4\pi}{3}$	B
12.	$\theta=20°,70°$	D
13.	$\frac{\sqrt{3}}{2}$	C
14.	$\frac{1}{\sqrt{1+x^2}}$	D
15.	3.96,5.46	A