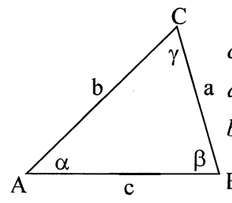
MA 15400

Fall 2013

Exam 3

LAW OF SINES:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$



LAW OF COSINES:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$
$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

Double Angle Formulas:

$$\sin(2u) = 2\sin u\cos u$$
$$\cos(2u) = \cos^2 u - \sin^2 u$$
$$\tan(2u) = \frac{2\tan u}{1-\tan^2 u}$$

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Angle Between Two Vectors:

$$\cos\theta = \frac{(\vec{a})\bullet(\vec{b})}{\|\vec{a}\|\|\vec{b}\|}$$

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Lessons 21-33, Sections 7.6, 8.1, 8.2, 8.3, and 8.4

1. Find the exact value of the expression.

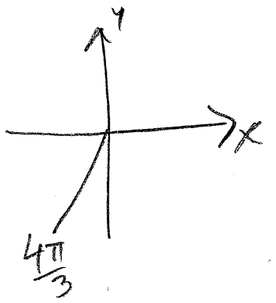
$$\tan^{-1}\left(\tan\left(\frac{4\pi}{3}\right)\right) = \text{TAn}^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$$

A. $\frac{-\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{\pi}{3}$

D. $\frac{5\pi}{3}$



E. None of the above

2. Write the expression as an algebraic expression in x for $x > 0$.

$$\sin\left(2\underbrace{\cos^{-1}x}_{\alpha}\right) \longrightarrow = \sin 2\alpha$$

$$= 2\sin\alpha\cos\alpha$$

$$= 2\left(\frac{\sqrt{1-x^2}}{1}\right)\left(\frac{x}{1}\right)$$

$$= 2x\sqrt{1-x^2}$$

A. $2x-2x^2$

B. $1-x^2$

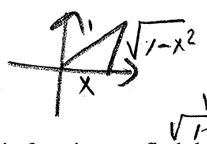
C. $-1+2x$

D. $2x\sqrt{1-x^2}$

E. None of the above

$\alpha = \cos^{-1}x$

$\cos\alpha = \frac{x}{1}$



$1^2 = x^2 + b^2$

$\sqrt{1-x^2} = b$

3. Use inverse trigonometric functions to find the solutions of the equation that are in the given interval, and approximate the solutions to four decimal places.

$$3\cos^2 x + 4\cos x - 5 = 0 \qquad [0, 2\pi)$$

A. 2.4756, 3.8076

B. 0.6660, 5.6172

C. 0.9048, 2.2368

D. 4.0464, 5.3784

E. None of the above

$$\cos x = \frac{-4 \pm \sqrt{16-4(3)(-5)}}{2(3)} = \frac{-4 \pm \sqrt{76}}{6}$$

$\cos x = \frac{-4+\sqrt{76}}{6}$ $\qquad$ $\cos x = \frac{-4-\sqrt{76}}{6}$

$\cos x = 0.7863$ QI QIV $\qquad$ $\cos x = -2.1196$

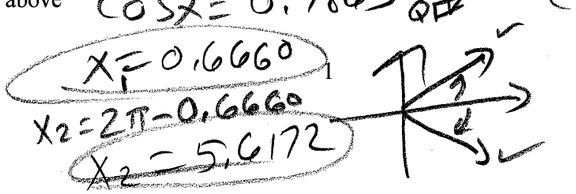
$x_1 = 0.6660$

$x_2 = 2\pi - 0.6660$

$x_2 = 5.6172$

No solution

$-2.1196 < -1$



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Lessons 21-33, Sections 7.6, 8.1, 8.2, 8.3, and 8.4

This would be a good time to check the mode on your calculator!

4. Given ΔABC with $\alpha = 97°$, $\gamma = 25°$, and $a = 21.5$. Find the values of sides b and c rounded to one decimal place.

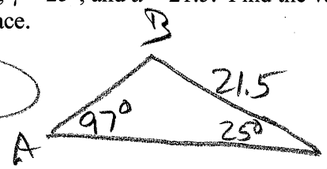
A. $b = 18.4$ and $c = 9.2$

B. $b = 18.9$ and $c = 8.1$

C. $b = 18.9$ and $c = 9.2$

D. $b = 18.4$ and $c = 8.1$

E. None of the above



$\beta = 180° - (97° + 25°)$

$\beta = 58°$

$\frac{\sin 97°}{21.5} = \frac{\sin 25°}{c}$ $\quad c = 9.1545$

$\frac{\sin 97°}{21.5} = \frac{\sin 58°}{b}$ $\quad b = 18.37$

5. A surveyor notes that the direction from point A to point B is N50°W and the direction from A to point C is S30°W. The distance from A to B is 275 yards, and the distance from B to C is 450 yards. Approximate the distance from A to C to the nearest whole yard.

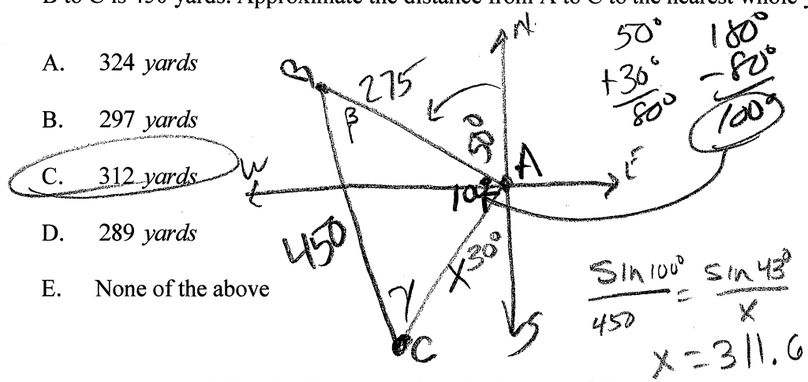
A. 324 *yards*

B. 297 *yards*

C. 312 *yards*

D. 289 *yards*

E. None of the above



$50° + 30° = 80°$

$180° - 80° = 100°$

$\frac{\sin 100°}{450} = \frac{\sin \gamma}{275}$

$\sin \gamma = 0.6018$

$\gamma = 37.00°$

$\beta = 180° - \alpha - \gamma$

$\beta = 43.00°$

$\frac{\sin 100°}{450} = \frac{\sin 43°}{x}$

$x = 311.6287$

6. An airplane flies 333 miles from point A in the direction 50° and then travels in the direction 200° for 222 miles. To the nearest mile, approximately how far is the airplane from A?

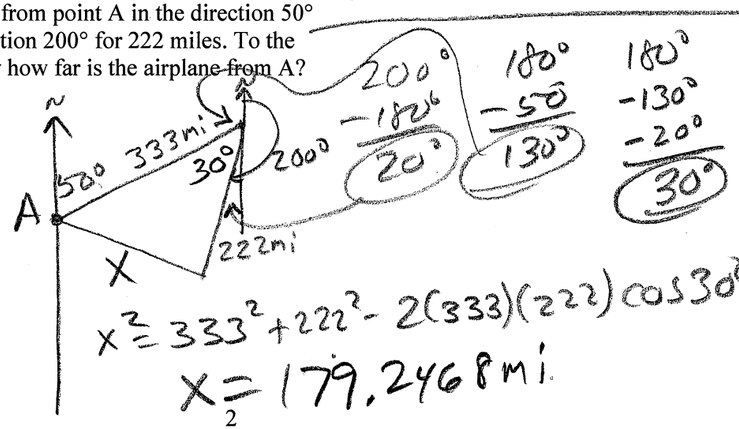
A. 267 *miles*

B. 198 *miles*

C. 248 *miles*

D. 179 *miles*

E. None of the above



$200° - 180° = 20°$

$180° - 50° = 130°$

$180° - 130° - 20° = 30°$

$x^2 = 333^2 + 222^2 - 2(333)(222)\cos 30°$

$x = 179.2468$ mi.

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Lessons 21-33, Sections 7.6, 8.1, 8.2, 8.3, and 8.4

7. Given ΔABC with $\alpha = 51°, a = 375$ *and* $c = 452$, which statement is true?

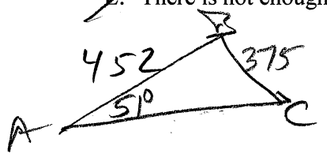
A. There exist two possible triangles and one of the values of $b = 153.2$.

B. There only exists one possible triangle and $\beta = 59.5°$

C. There exist two possible triangles and one of the values of $\gamma = 112.3°$.

D. There only exists one possible triangle and $b = 421.7$

E. There is not enough information to solve for the rest of the triangle.



Since $51° + 110.5° = 161.5°$, less than $180°$, there are two Δ's that exist.

$\beta_1 = 180° - (51° + 69.5°)$

$\beta_1 = 59.5°$

$\beta_2 = 180° - (110.5° + 51°)$

$\beta_2 = 18.5°$

$\frac{\sin 51°}{375} = \frac{\sin \gamma}{452}$

$\sin \gamma = 0.9367$

$\gamma_1 = 69.5°$ $\quad \gamma_2 = 180° - 69.5°$

$\gamma_2 = 110.49°$

$\frac{\sin 51°}{375} = \frac{\sin 18.5°}{b_2}$

$b_2 = 153.2$

$\frac{\sin 51°}{375} = \frac{\sin 59.5°}{b_1}$

$b_1 = 415.7$

8. Given $a = \langle 4, 6 \rangle$ and $b = \langle -5, 2 \rangle$ find $3a - 4b$.

A. $\langle -31, -18 \rangle$

B. $\langle -8, 36 \rangle$

C. $\langle 32, 10 \rangle$

D. $\langle 8, -36 \rangle$

E. None of the above

$3a = \langle 12, 18 \rangle$

$4b = -\langle -20, 8 \rangle$

$\langle 32, 10 \rangle$

9. Given vector $c = -8i + 4j$ find $\|c\|$ to the nearest tenth.

A. $\|c\| = 8.7$

B. $\|c\| = 8.1$

C. $\|c\| = 8.3$

D. $\|c\| = 8.5$

E. None of the above

$\|c\| = \sqrt{8^2 + 4^2} = \sqrt{64 + 16} = \sqrt{80}$

≈ 8.9

Extra Question

Smallest Angle From pos. x-AXIS

$\tan\theta = \frac{y}{x} = \frac{4}{-8}$ (<-,+> QII vector)

$\theta = \tan^{-1}(-\frac{1}{2}) + 180°$

$\theta = -26.6° + 180° = 153.4°$

10. The magnitudes and directions of two forces acting at a point P are…

$4.4 lb,\ 50°$

$9.7 lb,\ 165°$

Approximate the direction of the result vector to the nearest whole degree.

A. $\theta = 129°$

B. $\theta = 132°$

C. $\theta = 134°$

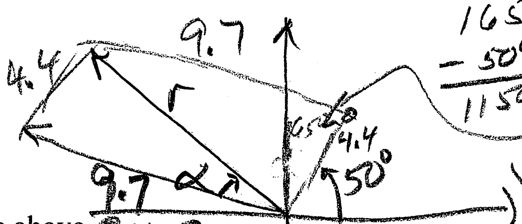
D. $\theta = 138°$

E. None of the above

Now use Law of Sines to find the largest angle in a Δ

I did not want to find the angle across from 9.7 because it might be obtuse.

$165° - 50° = 115°$ $180° - 115° = 65°$



$r^2 = 9.7^2 + 4.4^2 - 2(9.7)(4.4)\cos 65°$

$\|r\| = 8.7963$

$\frac{\sin\alpha}{4.4} = \frac{\sin 65°}{8.7963}$

$\alpha = 26.9584°$

$165° - 27° = 138°$

Vectors

a: $\langle 4.4\cos 50°, 4.4\sin 50° \rangle$

b: $\langle 9.7\cos 165°, 9.7\sin 165° \rangle$

r: $\langle -6.5412, 5.8811 \rangle$

$\|r\| = \sqrt{x^2+y^2} = 8.8$

$\tan\theta = \frac{y}{x} = \frac{5.9}{-6.5}$

$\theta = \tan^{-1}(-0.90) + 180°$

$\theta = 138°$

11. Find a vector that has the same direction as $\langle -6, 8 \rangle$ and five times its magnitude.

A. $\langle 30, -40 \rangle$

B. $\langle 3, -4 \rangle$

C. $\langle -30, 40 \rangle$

D. $\langle -3, 4 \rangle$

E. None of the above

$5\langle -6, 8 \rangle$

$\langle -30, 40 \rangle$

12. Find the angle between the two vectors $a = \langle -7, -6 \rangle, b = \langle -2, 10 \rangle$.
Round to the nearest tenth of a degree.

A. 121.5°

B. 119.3°

C. 122.6°

D. 113.9°

E. None of the above

$a \cdot b = (-7)(-2) + (-6)(10)$

$= +14 - 60$

$= -46$

$\|a\| = \sqrt{7^2+6^2} = \sqrt{85}$

$\|b\| = \sqrt{2^2+10^2} = \sqrt{104}$

$\cos\theta = \frac{-46}{\sqrt{85}\sqrt{104}}$

$\cos\theta = -0.4893$

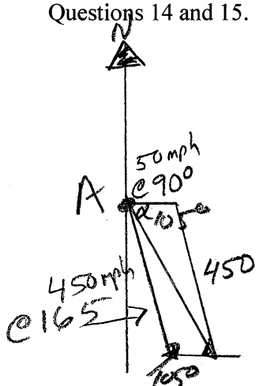
$\theta = 119.2914°$

13. Determine m such that the two vectors $c=\langle 2m,-4\rangle, d=\langle 5,6\rangle$ are orthogonal.

A. $m=3.75$
B. $m=-2.4$
C. $m=-3.75$
D. $m=2.4$
E. None of the above

$C\cdot d=(2m)(5)+(-4)(6)$
$=10m-24$
$10m-24=0$
$10m=24$
$m=2.4$

Questions 14 and 15. An airplane is flying in the direction 165° with airspeed of 450 mph, and a 50 mph wind is blowing directly **from** the west.



$165°-90°=75°$
$180°-75°=105°$

Vectors $P+W=R$
$P:\langle 450\cos 165°, 450\sin 165°\rangle$
$W:\langle 50\cos 90°, 50\sin 90°\rangle$
$R:\langle -434.6666, 166.4686\rangle$

14. Approximate the Ground Speed of the airplane to the nearest mph.

A. 465 *mph*
B. 441 *mph*
C. 447 *mph*
D. 471 *mph*
E. None of the above

$r^2=450^2+50^2-2(450)(50)\cos 105°$
$\|r\|\approx 465.4534$ mph

Vector: $\|r\|=\sqrt{x^2+y^2}=\sqrt{(434.7)^2+(166.5)^2}$
$\|r\|=465.4534$ mph

15. Approximate the True Course of the airplane to the nearest whole degree.

A. 157°
B. 159°
C. 151°
D. 153°
E. None of the above

$\frac{\sin\alpha}{450}=\frac{\sin 105°}{465.4534}$
$\sin\alpha=0.9339$
$\alpha=69.0°$
$69°+90°=159°$

Vectors
$\tan\theta=\frac{y}{x}=\frac{166.5}{-435.7}$
$\theta=\tan^{-1}(-0.38)+180°$
$\theta=-21°+180°$
$\theta=159°$

Exam 3 Answers

1.	C	$\frac{\pi}{3}$
2.	D	$2x\sqrt{1-x^2}$
3.	B	$0.6660, 5.6172$
4.	A	$b = 18.4$ and $c = 9.2$
5.	C	312 *yards*
6.	D	179 *miles*
7.	A	There exist two possible triangles and one of the values of $b = 153.2$.
8.	C	$\langle 32, 10 \rangle$
9.	E	$\|c\| = 8.9$
10.	D	$\theta = 138°$
11.	C	$\langle -30, 40 \rangle$
12.	B	$119.3°$
13.	D	$m = 2.4$
14.	A	465 *mph*
15.	B	$159°$