MA 15400

Fall 2014

Exam 3

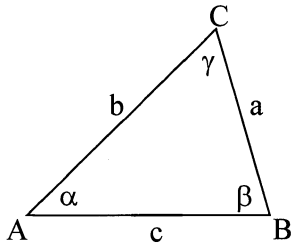
Solutions

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

$$a^2 = c^2 + b^2 - 2cb\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin(2u) = 2\sin u\cos u$$

$$\cos(2u) = \cos^2 u - \sin^2 u$$

$$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$$

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Lessons 21-32, Covers Sections 7.6, 8.1, 8.2, and 8.3

1. Find the exact value of $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) =$ $\frac{5\pi}{6}$

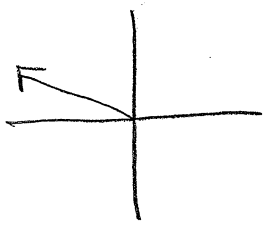
A. $\frac{-\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{-5\pi}{6}$

D. $\frac{\pi}{6}$

E. None of the above



2. Find the exact value of $\arcsin\left(\sin\frac{4\pi}{3}\right)$.

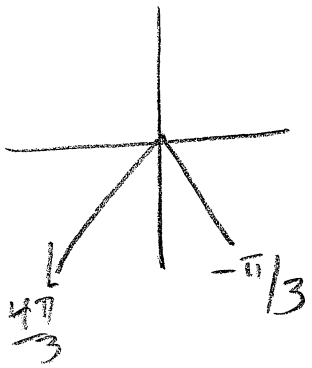
A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{-\pi}{3}$

D. $\frac{5\pi}{3}$

E. $\frac{4\pi}{3}$ (Not the answer! Do not pick this.)



3. Find the solutions of the equation $5\tan^2 t + 3\tan t - 9 = 0$ in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Round to nearest 0.0001 radians. **Check the mode on your calculator!**

A. $-1.0325, 0.8214$

B. $-1.7266, 0.9266$

C. $-1.0458, 0.7473$

D. $-1.6748, 1.0748$

E. None of the above

$\tan t = \frac{-3 \pm \sqrt{9-4(5)(-9)}}{2(5)} = \frac{-3 \pm \sqrt{189}}{10}$

$\tan t = \frac{-3+\sqrt{189}}{10}$ $\left(\tan t = \frac{-3-\sqrt{189}}{10}\right)$

$\tan t = 1.0748$

$t = \tan^{-1}(1.0748)$

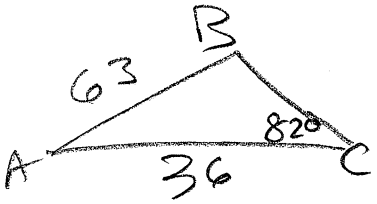
$t = 0.8214$

$\tan t = -1.6748$

$t = \tan^{-1}(-1.6748)$

$t = -1.0325$

4. Find the perimeter of ΔABC, given angle $\gamma = 82°$, side $b = 36$, and side $c = 63$. Round to the nearest whole number. **Check the mode on your calculator!**



A. 136
B. 129
C. 161
D. 156
E. None of the above

$\frac{\sin 82°}{63} = \frac{\sin B}{36}$

$\sin B = \frac{36 \sin 82°}{63}$

$\sin B = 0.5659$

$B = 34.4626°$

$\alpha = 180° - (\gamma + B)$

$\alpha = 63.5374°$

$\frac{\sin 63.5°}{a} = \frac{\sin 82°}{63}$

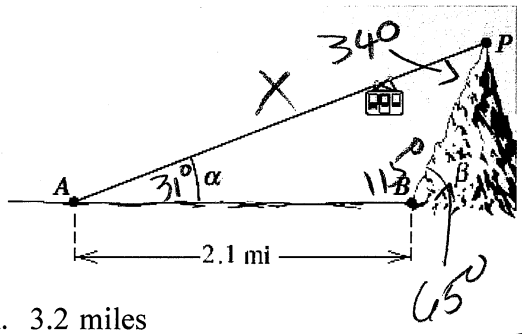
$a = \frac{63 \sin 63.5°}{\sin 82°}$

$a = 56.9535$

$P = a + b + c$

$P = 155.9535$

5. As shown in the figure below, a cable car carries passengers from a point A, which is 2.1 miles from a point B at the base of a mountain, to a point P at the top of the mountain. The angles of elevation of P from A and B are $\alpha = 31°$ and $\beta = 65°$, respectively. Find the distance from Point A to Point P to the nearest tenth of a mile.



A. 3.2 miles
B. 3.0 miles
C. 3.6 miles
D. 3.4 miles
E. None of the above

$180° - 65° = 115°$

$31° + 115° = 146°$

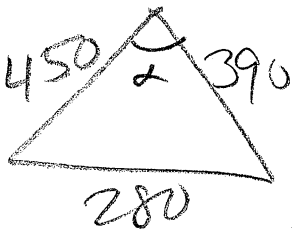
$180° - 146° = 34°$

$\frac{\sin 34°}{2.1} = \frac{\sin 115°}{X}$

$X = \frac{2.1 \sin 115°}{\sin 34°}$

$X = 3.404$

6. A triangular plot of land has sides of lengths 450 feet, 390 feet, and 280 feet. Approximate the smallest angle between the sides to one decimal place.



A. 27.9°
B. 38.1°
C. 23.4°
D. 32.7°
E. None of the above

$280^2 = 390^2 + 450^2 - 2(390)(450)\cos\alpha$

$-276200 = -351000\cos\alpha$

$\cos\alpha = 0.7869$

$\alpha = \cos^{-1}(0.7869)$

$\alpha = 38.1038°$

7. A ship leaves port at 1:00 pm and sails in the direction N40°E at a rate of 50 mph. At 2:00 pm a second ship leaves the same port and sails in the direction N35°W at a rate of 25 mph. To the nearest mile, how far apart are the two ships at 4:00 pm?

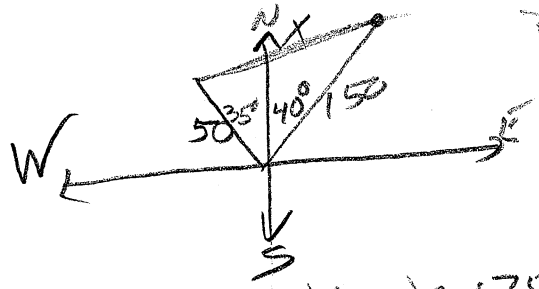
A. 163 miles

B. 170 miles

C. 145 miles

D. 158 miles

E. None of the above



$D = rt$

$D_1 = 50(3) = 150$

$D_2 = 25(2) = 50$

$35° + 40° = 75°$

$x^2 = 50^2 + 150^2 - 2(50)(150)\cos 75°$

$x = 145.3194$

8. Given $a = \langle 5, -7 \rangle$ and $b = \langle 6, 3 \rangle$, find $4a + 5b$.

A. $\langle 50, -13 \rangle$

B. $\langle -10, -43 \rangle$

C. $\langle 10, -43 \rangle$

D. $\langle -50, 13 \rangle$

E. None of the above

$4a = \langle 20, -28 \rangle$

$5b = \langle 30, 15 \rangle$

$+$

$\langle 50, -13 \rangle$

9. The vectors a and b represent two forces acting at the same point, and θ is the smallest positive angle between a and b. Approximate the magnitude of the resultant force to one decimal place.

$$\|a\| = 8.2\,lb, \quad \|b\| = 12.5\,lb, \quad \theta = 60°$$

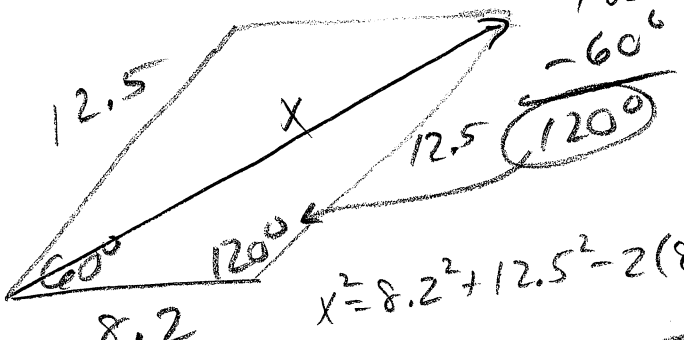
A. $\|r\| = 11.0\ lb$

B. $\|r\| = 18.1\ lb$

C. $\|r\| = 14.7\ lb$

D. $\|r\| = 17.3\ lb$

E. None of the above



$x^2 = 8.2^2 + 12.5^2 - 2(8.2)(12.5)\cos 120°$

$x = 18.055$

10. Find a vector that has the same direction as $5i-8j$ and 6 times the magnitude.

A. $\frac{-30}{\sqrt{89}}i+\frac{48}{\sqrt{89}}j$

B. $-30i+48j$

C. $\frac{30}{\sqrt{89}}i-\frac{48}{\sqrt{89}}j$

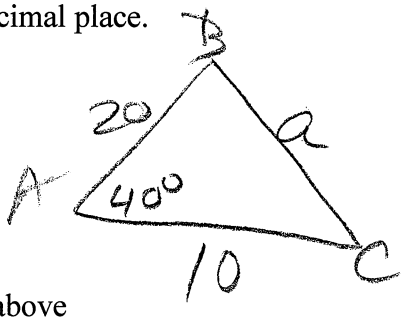
D. $30i-48j$

E. None of the above

$6(5i-8j)$

$30i-48j$

11. Find side a of ΔABC given $\alpha=40°$, $b=10$, and $c=20$.
Round to one decimal place.



A. 13.5

B. 13.9

C. 14.4

D. 14.8

E. None of the above

$a^2=10^2+20^2-2(10)(20)\cos 40°$

$a=13.913$

For Questions 12 and 13, use vector $a=\langle 7,13\rangle$ and round to one decimal place.

12. What is the magnitude of vector a?

A. $\|a\|=14.8$

B. $\|a\|=14.5$

C. $\|a\|=15.1$

D. $\|a\|=15.4$

E. None of the above

$\|a\|=\sqrt{7^2+13^2}$

$\|a\|=14.764$

13. What is the smallest positive angle, θ, between the positive x-axis and vector a?

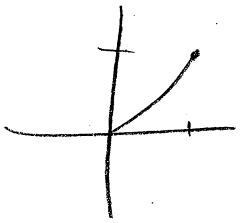
A. $\theta=28.3°$

B $\theta=36.4°$

C. $\theta=61.7°$

D. $\theta=72.5°$

E. None of the above



$\tan\theta=\frac{y}{x}=\frac{13}{7}$

$\theta=\tan(\frac{13}{7})$

$\theta=61.6992°$

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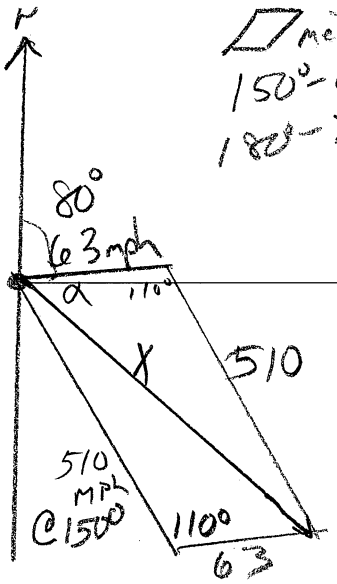
Lessons 21-32, Covers Sections 7.6, 8.1, 8.2, and 8.3

Questions 14 and 15: An airplane with an airspeed of 510 mph is flying in the direction 150° and a 63 mph wind is blowing in the direction of 80°.



Parallelogram method

$150° - 80° = 70°$

$180° - 70° = 110°$

Vector method

$P \langle 510\cos 150°, 510\sin 150° \rangle$

$+ W \langle 63\cos 80°, 63\sin 80° \rangle$

$r \langle -430.7331, 317.0429 \rangle$

QII vector $\langle +, - \rangle$

need to $+180°$

to $\tan^{-1}$

14. What is the ground speed of the plane? Round to the nearest whole number.

A. 535 *mph*

B. 476 *mph*

C. 549 *mph*

D. 498 *mph*

E. None of the above

Parallelogram method

$X^2 = 510^2 + 63^2 - 2(510)(63)\cos 110°$

$X = 534.8338$ mph

Vector method

$\|r\| = \sqrt{x^2 + y^2} = \sqrt{(-430.7)^2 + (317.0)^2}$

$\|r\| = 534.8338$

15. What is the true course of the plane? Round to the nearest whole degree.

A. 135°

B. 141°

C. 138°

D. 144°

E. None of the above

Parallelogram method

$\frac{\sin 110°}{534.8338} = \frac{\sin\alpha}{510}$

$\sin\alpha = \frac{510\sin 110°}{534.8338}$

$\alpha = 63.6°$

True Course $= 80° + 63.6°$

$= 143.6449°$

Vector method

$\tan\theta = \frac{y}{x} = \frac{317.0}{-430.7}$

$\theta = \tan^{-1}(-0.7361) + 180$

$\theta = -36.4° + 180°$

$\theta = 143.6449°$

Questions	Answers	Letters
1.	$\frac{5\pi}{6}$	B
2.	$\frac{-\pi}{3}$	C
3.	$-1.0325, 0.8214$	A
4.	156	D
5.	3.4 miles	D
6.	$38.1°$	B
7.	145 miles	C
8.	$\langle 50, -13 \rangle$	A
9.	$\|\|r\|\| = 18.1\ lb$	B
10.	$30i - 48j$	D
11.	13.9	B
12.	$\|\|a\|\| = 14.8$	A
13.	$\theta = 61.7°$	C
14.	$535\ mph$	A
15.	$144°$	D