MA 15400

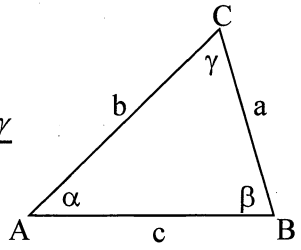
Spring 2013

Exam 3

Also, 3 Questions from EXAM 2

LAW OF SINES

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$



LAW OF COSINES

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

ANGLE BETWEEN TWO VECTORS:

$$\cos\theta = \frac{(\vec{a})\bullet(\vec{b})}{\|\vec{a}\|\,\|\vec{b}\|}$$

13. Find the exact value of the expression whenever it is defined.

$$\cos\left(\sin^{-1}\left(\frac{-1}{2}\right)\right)$$

A. $\frac{-\sqrt{3}}{2}$

B. $\frac{1}{2}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{-1}{2}$

E. None of the above

$\cos(-\pi/6)$

$\sqrt{3}/2$

14. Write the expression as an algebraic expression in x for $x > 0$.

$$\cos(\tan^{-1} x)$$

A. $\frac{x}{\sqrt{1+x^2}}$

B. $\frac{1}{1+x}$

C. $\frac{x}{1+x}$

D. $\frac{1}{\sqrt{1+x^2}}$

E. None of the above

$\alpha = \tan^{-1} x$

$\tan\alpha = \frac{x}{1}$

$c^2 = 1^2 + x^2$

$c = \sqrt{1+x^2}$

$= \frac{1}{\sqrt{1+x^2}}$

15. Approximate the solutions of the equation, to two decimals, that are in the given interval.

$$\sin^2 x - 2\sin x - 2 = 0;\ [0, 2\pi)$$

A. 3.96,5.46

B. 2.36,3.18

C. 2.73,3.87

D. 3.57,5.14

E. None of the above

$\sin x = \frac{2 \pm \sqrt{4-4(1)(-2)}}{2(1)}$

$\sin x = \frac{2 \pm \sqrt{12}}{2}$

$\sin x = \frac{2 \pm 2\sqrt{3}}{2}$

$\sin x = 1 \pm \sqrt{3}$

$\sin x = 1-\sqrt{3}$ $\sin x = 1+\sqrt{3}$

$\sin x = -0.7321$

$x = -0.8213$

QIII + QIV

$x_1 = \pi + 0.8213$

$x_1 = 3.96$

$x_2 = 2\pi - 0.8213$

$x_2 = 5.46$

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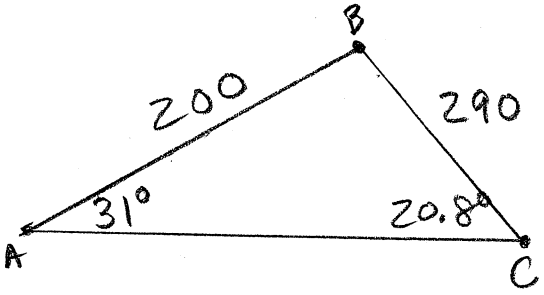
Lessons 24-33, All of Sections 8.1, 8.2, 8.3 and 8.4

Say out loud, "I am smart, I am good looking, and people love me!"

1. Find the perimeter of ΔABC, with $\alpha = 31°, a = 290\ cm,$ *and* $c = 200\ cm,$ to the nearest tenth.



A. 932.5 *cm*

B. 875.4 *cm*

C. 754.3 *cm*

D. 805.9 *cm*

E. None of the above

$\frac{\sin 31°}{290} = \frac{\sin \gamma}{200}$

$\sin \gamma = 0.3552$

$\gamma = 20.8°, \gamma = 159.2°$

$\beta = 180° - 20.8° - 31.0$

$+ 31.0$

$190.2°$

$128.2°$

only one Tri. Angle

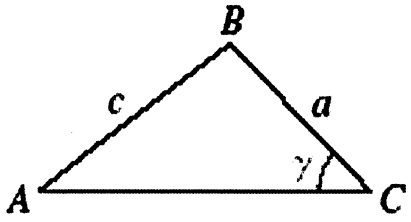
$P = 200 + 290 + 442.5$

$P = 932.5\ cm$

$\frac{\sin 31°}{290} = \frac{\sin 128.2°}{b}$

$b = 442.5$

2. Given sides c and a, and angle γ of ΔABC, what angle or side would you find next, and what would you use to find it?



A. Angle β, Law of Sines

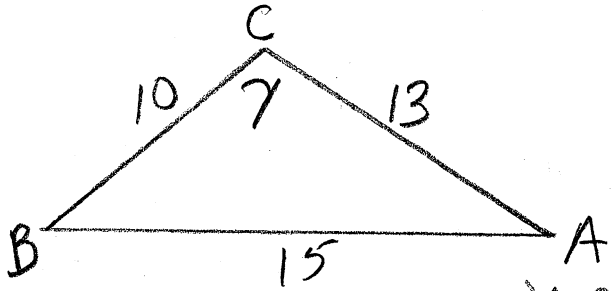
B. Side b, Law of Cosines

C. Angle α, Law of Sines

D. Side b, Law of Sines

E. Angle α, Law of Cosines

3. Given ΔABC, with $a = 10, b = 13,$ and $c = 15,$ find angle γ to the nearest tenth of a degree.



A. $\gamma = 99.7°$

B. $\gamma = 58.7°$

C. $\gamma = 121.3°$

D. $\gamma = 80.3°$

E. None of the above.

$15^2 = 10^2 + 13^2 - 2(10)(13)\cos\gamma$

$\frac{225 - 100 - 169}{-260} = \cos\gamma$

$\cos\gamma = 0.1692$

$\gamma = 80.3°$

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Lessons 24-33, All of Sections 8.1, 8.2, 8.3 and 8.4

4. To find the distance between two points A and B, that lie on opposite sides of a river, a surveyor picks a point C that is 290 yards from Point A, and on the same side of the river as Point A. The surveyor determines that the measures of $\angle BAC = 60°$ and $\angle ACB = 54°$. Approximate the distance between A and B. Round your answer to the nearest yard.

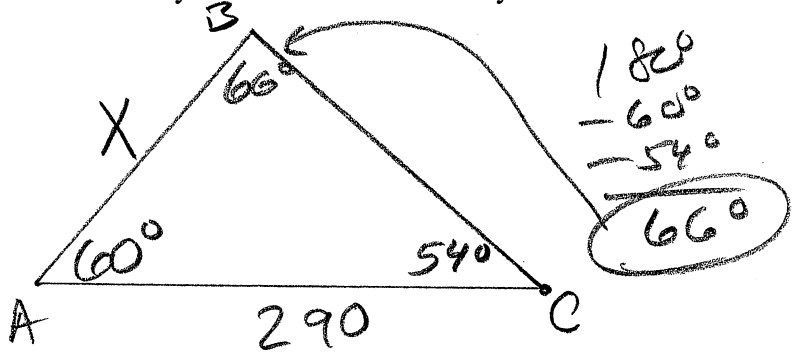
A. 264 *yards*

B. 271 *yards*

C. 260 *yards*

D. 274 *yards*

E. None of the above



180 − 60 − 54 = 66°

257 yds

$\frac{\sin 66°}{290} = \frac{\sin 54°}{x}$

$x = 256.8$

5. The given information about ΔABC, $\gamma = 34°, c = 20$ *and* $b = 30$, creates different two triangles. To the nearest whole degree, find the smaller of the two values of angle α.

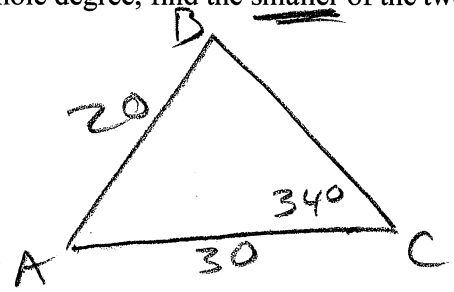
A. 61°

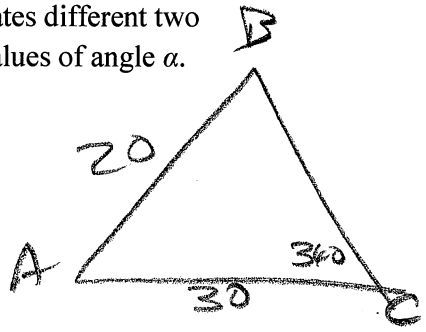
B. 23°

C. 57°

D. 19°

E. None of the above





$\frac{\sin 34°}{20} = \frac{\sin B}{30}$

$\sin B = \frac{30 \sin 34°}{20}$

$\sin B = 0.8388$

$B_1 = 57.00°$

$\alpha_1 = 89°$

$\gamma = 34°$

$B_2 = 180° - 57° = 123.00$

$\alpha_2 = 180° - 123° - 34° = 23°$

$\alpha_2 = 23°$

6. An airplane is flying in the direction 130°, with airspeed of 400 miles per hour, and a 50 mile per hour wind is blowing in the direction 20°. Approximate the ground speed of the airplane to the nearest mile per hour.

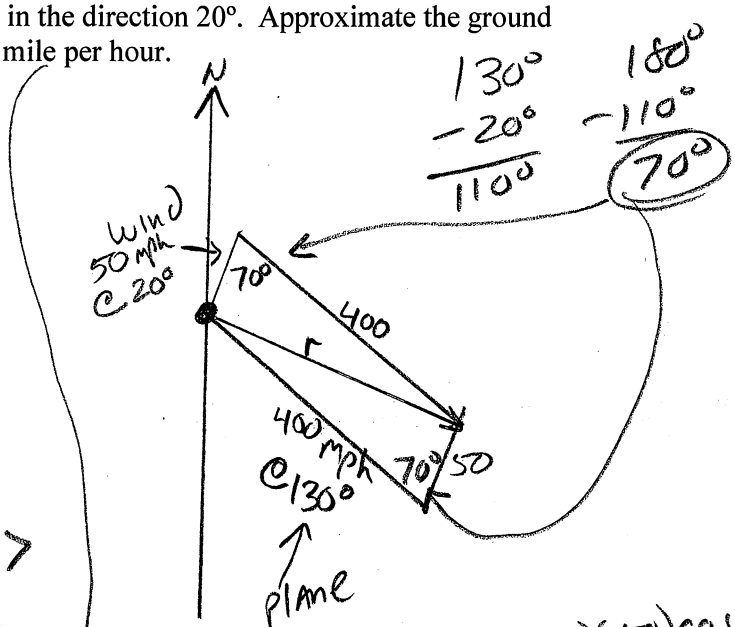
A. 434 *miles per hour*

B. 353 *miles per hour*

C. 420 *miles per hour*

D. 386 *miles per hour*

E. None of the above.



Vector: $P+w=r$

$P: \langle 400\cos 130°, 400\sin 130° \rangle$

$+w: \langle 50\cos 20°, 50\sin 20° \rangle$

$r: \langle -210.1304, 323.5188 \rangle$

$\|r\| = \sqrt{x^2+y^2} = 385.8$

$r^2 = 400^2 + 50^2 - 2(400)(50)\cos 70°$

$\|r\| \approx 385.8$ mph

7. Given vectors $a = \langle 3, -7 \rangle$ and $b = \langle 8, 5 \rangle$, find $3a - 2b$.

A. $\langle 25, -11 \rangle$

B. $\langle 30, 1 \rangle$

C. $\langle -7, -31 \rangle$

D. $\langle -18, 29 \rangle$

E. None of the above.

$3a = \langle 9, -21 \rangle$

$2b = \langle 16, 10 \rangle$

$\langle -7, -31 \rangle$

8. Given vector $a = -6i - 5j$, find $\|a\|$, to the nearest tenth, and the smallest positive angle θ from the positive x-axis to vector a to the nearest 0.1°.

A. $\|a\| = 7.8, \theta = 219.8°$

B. $\|a\| = 4.6, \theta = 230.2°$

C. $\|a\| = 7.8, \theta = 230.2°$

D. $\|a\| = 4.6, \theta = 219.8°$

E. None of the above.

$\|a\| = \sqrt{6^2+5^2} = \sqrt{61} \approx 7.8$

$\text{Tan}\,\theta = \frac{y}{x} = \frac{-5}{-6}$ $\angle -6, -5$ is in Q III

$\theta = \text{Tan}^{-1}(\frac{5}{6}) + 180°$ $+180°$

$\theta = 39.8° + 180°$

$\theta = 219.8°$

9. The vectors $\|a\| = 7.8$ lb and $\|b\| = 4.3$ lb represent two forces acting at the same point, and $\theta = 78°$ is the smallest positive angle between a and b. Approximate the magnitude of the resultant force to the nearest tenth.

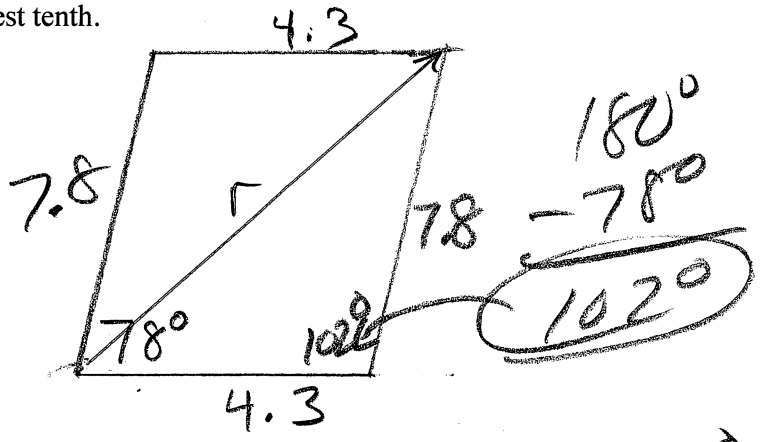
A. $\|r\| = 8.1$ lb

B. $\|r\| = 9.7$ lb

C. $\|r\| = 8.5$ lb

D. $\|r\| = 9.3$ lb

E. None of the above.



$180° - 78° = 102°$

$r^2 = 4.3^2 + 7.8^2 - 2(4.3)(7.8)\cos 102°$

$\|r\| = 9.6580$

10. Approximate, to the nearest tenth, the horizontal and vertical components of the vectors that is described.

A quarterback releases a football with a speed of 41 ft/sec at an angle of 56° to the horizontal.

A. Horizontal Component: 21.7 ft/sec
Vertical Component: 35.2 ft/sec

B. Horizontal Component: 34.0 ft/sec
Vertical Component: 22.9 ft/sec

C. Horizontal Component: 35.2 ft/sec
Vertical Component: 21.7 ft/sec

D. Horizontal Component: 22.9 ft/sec
Vertical Component: 34.0 ft/sec

E. None of the above.

Horz: $41\cos 56^\circ$
22.9 ft/sec

Vert: $41\sin 56^\circ$
34.0 ft/sec

$\langle 22.9, 34.0 \rangle$

11. Find the vector that has three times the magnitude and is in the same direction as the given vector. $a = 3i - 7j$

A. $9i - 21j$

B. $\frac{-9}{\sqrt{52}}i + \frac{21}{\sqrt{52}}j$

C. $-9i + 21j$

D. $\frac{9}{\sqrt{52}}i - \frac{21}{\sqrt{52}}j$

E. None of the above.

$3a = 3(3i - 7j)$
$= 9i - 21j$

12. $F_1\langle 4,-5\rangle, F_2\langle -3,-8\rangle$, and $F_3\langle -7,20\rangle$ are three forces acting at the same point, find the additional force **G** that needs to be added in order for there to be equilibrium.

A. $G=\langle -6,7\rangle$

B. $G=\langle -8,12\rangle$

C. $G=\langle 6,-7\rangle$

D. $G=\langle 8,-12\rangle$

E. None of the above.

$F_{net}=\langle 4+(-3)+(-7),(-5)+(-8)+20\rangle$

$F_{net}=\langle -6,7\rangle$

$G=+\langle 6,-7\rangle$ Answer

$\langle 0,0\rangle \leftarrow$ Equilibrium

13. Find the angle, to the nearest 0.1°, between the vectors, $a=\langle 9,7\rangle$ and $b=\langle 4,-12\rangle$

A. 56.3°

B. 109.4°

C. 70.6°

D. 123.6°

E. None of the above.

$a\cdot b=(9)(4)+(7)(-12)=-48$

$\|a\|=\sqrt{9^2+7^2}=\sqrt{130}$

$\|b\|=\sqrt{4^2+12^2}=\sqrt{160}$

$\cos\theta=\dfrac{-48}{\sqrt{130}\sqrt{160}}=\dfrac{-48}{\sqrt{20800}}=-0.3328$

$\theta=\cos^{-1}(-0.3328)$

$\theta=109.4^\circ$

14. Determine m such that the two vectors $a = -4i + 7j$ and $b = 7i + 2mj$ are orthogonal.

A. $m = 2$

B. $m = \frac{1}{2}$

C. $m = -2$

D. $m = \frac{-1}{2}$

E. None of the above.

$a \cdot b = (-4)(7) + (7)(2m)$

$= -28 + 14m$

$-28 + 14m = 0$

$14m = 28$

$m = 2$

15. The magnitude and direction of the two forces acting at a point P are:

$a = 8.4$ lb @ 100° and $b = 4.2$ lb @ 225°

Approximate, to the nearest 0.1°, the direction of the resultant vector.

A. $\theta = 135.1°$

B. $\theta = 115.4°$

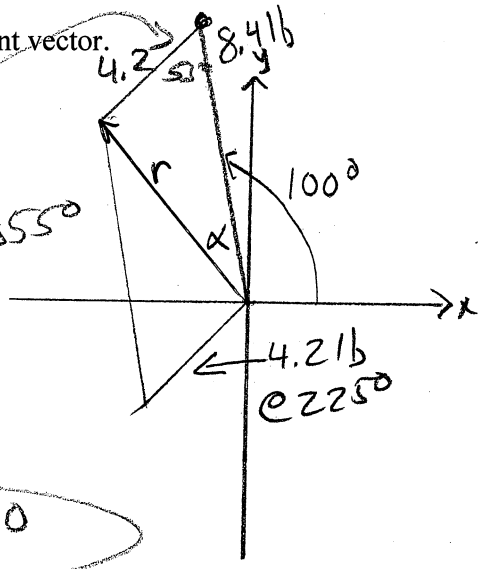
C. $\theta = 119.5°$

D. $\theta = 129.9°$

E. None of the above.

$225° - 180° = 45°$

$100° - 45° = 55°$



$r^2 = 8.4^2 + 4.2^2 - 2(8.4)(4.2)\cos 55°$

$\|r\| = 6.9086$

$\frac{\sin\alpha}{4.2} = \frac{\sin 55°}{6.9086}$

$\alpha = 29.9°$

$100° + 29.9° = 129.9°$

Vector: $a: \langle 8.4\cos 100°, 8.4\sin 100° \rangle$

$+ b: \langle 4.2\cos 225°, 4.2\sin 225° \rangle$

$r: \langle -4.4285, 5.3025 \rangle$

Also: $\|r\| = \sqrt{x^2 + y^2} = 6.9086$

$\text{Tan}\,\theta = \frac{y}{x} = \frac{5.3}{-4.4}$

$\theta = \text{Tan}^{-1}(-1.2) + 180°$

$\theta = -50.1° + 180°$

$\theta = 129.9°$

Question	Exam 3 Answers	
1.	$932.5\,cm$	A
2.	Angle α, Law of Sines	C
3.	$\gamma = 80.3°$	D
4.	257 yards	E
5.	23°	B
6.	386 *miles per hour*	D
7.	$\langle -7, -31 \rangle$	C
8.	$\|\|a\|\| = 7.8, \theta = 219.8°$	A
9.	$\|\|r\|\| = 9.7$ lb	B
10.	Horizontal Component: 22.9 ft/sec Vertical Component: 34.0 ft/sec	D
11.	$9i - 21j$	A
12.	$G = \langle 6, -7 \rangle$	C
13.	109.4°	B
14.	$m = 2$	A
15.	$\theta = 129.9°$	D