MA 15400

Spring 2014

Solutions

Exam 3

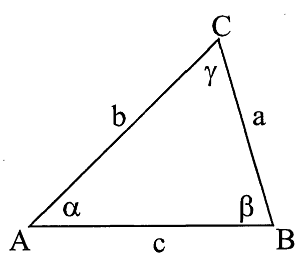
$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

$$a^2 = c^2 + b^2 - 2cb\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$\cos\theta = \frac{(\vec{a})\bullet(\vec{b})}{\|\vec{a}\|\,\|\vec{b}\|}$$



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin(2u) = 2\sin u\cos u$$

$$\cos(2u) = \cos^2 u - \sin^2 u$$

$$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$$

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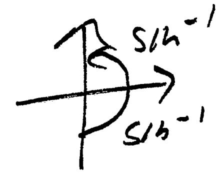
Lessons 21-33, All of Sections 7.6, 8.1, 8.2, 8.3, and 8.4

1. Find the exact value of the expression. $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$.



A. $\frac{-\pi}{3}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{4\pi}{3}$

E. $\frac{5\pi}{3}$ (Not the answer. I am NOT joking, do not pick this.)

$\theta_R = \frac{\pi}{3}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\frac{5\pi}{3}$ is in Q IV, sine is negative

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$-\pi/3$

2. Find the exact value of the expression. $\cos\left(\underbrace{2\arcsin\frac{5}{13}}_{\alpha}\right) = \cos 2\alpha$

A. $\frac{120}{169}$

B. $\frac{-119}{169}$

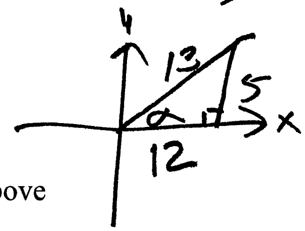
C. $\frac{-120}{169}$

D. $\frac{119}{169}$

E. None of the above

$\alpha = \sin^{-1}\frac{5}{13}$

$\sin\alpha = \frac{5}{13}$



$= \cos^2\alpha - \sin^2\alpha$

$= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{144}{169} - \frac{25}{169} = \frac{119}{169}$

3. Find the solutions of the equation that are in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ rounded to four decimal places: $2\tan^2 t - 7\tan t + 4 = 0$

A. $1.3148, -0.6847$

B. $2.7808, 0.7192$

C. $1.2256, 0.6235$

D. $3.8206, -0.8165$

E. None of the above

$a = 2 \quad b = -7 \quad c = 4$

$\tan t = \frac{7 \pm \sqrt{49 - 4(2)(4)}}{2(2)} = \frac{7 \pm \sqrt{17}}{4}$

$\tan t = \frac{7 + \sqrt{17}}{4}$ $\qquad$ $\tan t = \frac{7 - \sqrt{17}}{4}$

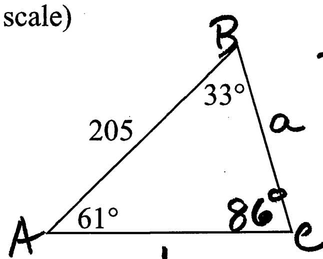
$\tan t = 2.7808$ $\qquad$ $\tan t = 0.7192$

$t = 1.2256$ $\qquad$ $t = 0.6235$

Lessons 21-33, All of Sections 7.6, 8.1, 8.2, 8.3, and 8.4

This would be a good time to check the mode of your calculator.

4. Approximate the perimeter of the given triangle to the nearest whole number. (Not drawn to scale)



A. 487

B. 465

C. 478

D. 497

E. None of the above

$\gamma = 180^\circ - (61^\circ + 33^\circ)$

$\gamma = 86^\circ$

$\frac{\sin 86^\circ}{205} = \frac{\sin 33^\circ}{b}$

$b = 111.9$

$\frac{\sin 86^\circ}{205} = \frac{\sin 61^\circ}{a}$

$a = 179.7$

$P = a + b + c = 496.6585$

5. The following information about $\triangle ABC$ creates two distinct triangles. Solve for both triangles and find the larger value of the two angle β.

$$\gamma = 68^\circ, b = 14, c = 13$$

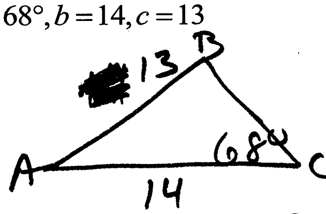
A. Larger $\beta = 89.9^\circ$

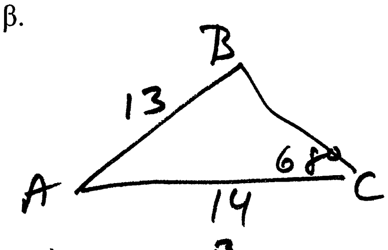
B. Larger $\beta = 93.1^\circ$

C. Larger $\beta = 96.2^\circ$

D. Larger $\beta = 84.4^\circ$

E. None of the above





$\frac{\sin 68^\circ}{13} = \frac{\sin \beta}{14}$

$\sin\beta = \frac{14 \sin 68^\circ}{13}$

$\sin\beta = 0.9985$

$\beta_1 = 86.9^\circ$ $\beta_2 = 180^\circ - 86.9^\circ = 93.1^\circ$

6. Given the following information about $\triangle ABC$, find the value of side c to the nearest tenth.

$$\gamma = 17^\circ, b = 81, a = 62$$

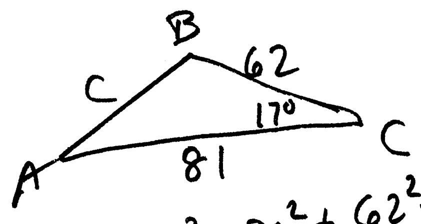
A. $c = 27.8$

B. $c = 26.5$

C. $c = 27.1$

D. $c = 28.7$

E. None of the above

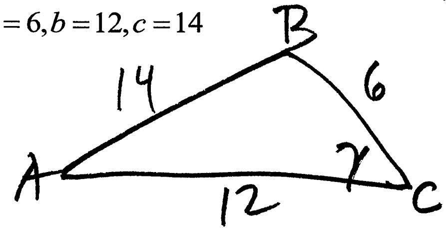


$c^2 = 81^2 + 62^2 - 2(81)(62)\cos 17^\circ$

$c = 28.2821$

7. Given the following information about $\triangle ABC$, find the value of the **largest angle** in the triangle to the nearest tenth.

$a = 6, b = 12, c = 14$



A. 83.6°
B. 77.2°
C. 96.4°
D. 102.8°
E. None of the above

$14^2 = 12^2 + 6^2 - 2(12)(6)\cos\gamma$
$196 = 144 + 36 - 144\cos\gamma$
$196 = 180 - 144\cos\gamma$
$16 = -144\cos\gamma$
$\frac{16}{-144} = \cos\gamma$
$\gamma = \cos^{-1}(-0.1111)$
$\gamma = 96.3794^\circ$

8. Given vectors a and b, find $4a + 5b$.

$a = \langle 2, -5\rangle, b = \langle 3, 1\rangle$

A. $\langle -14, 19\rangle$
B. $\langle 22, -21\rangle$
C. $\langle -15, 13\rangle$
D. $\langle 23, -15\rangle$
E. None of the above

$4a = 4\langle 2, -5\rangle = \langle 8, -20\rangle$
$5b = 5\langle 3, 1\rangle = +\langle 15, 5\rangle$
$4a + 5b = \langle 23, -15\rangle$

9. Find a vector of magnitude 6 that has the opposite direction of vector $b = -4i + 7j$

A. $\frac{24}{\sqrt{65}}i - \frac{42}{\sqrt{65}}j$
B. $-24i + 42j$
C. $\frac{-24}{\sqrt{65}}i + \frac{42}{\sqrt{65}}j$
D. $24i - 42j$
E. None of the above

1. Find u
2. $-6u$

Find u
1. $\|b\| = \sqrt{4^2 + 7^2}$
$\|b\| = \sqrt{65}$
$u = \frac{1}{\sqrt{65}}(-4i + 7j)$
$u = -\frac{4}{\sqrt{65}}i + \frac{7}{\sqrt{65}}j$

2. $-6u$
$-6u = -6\left(\frac{-4}{\sqrt{65}}i + \frac{7}{\sqrt{65}}j\right)$
$= \frac{24}{\sqrt{65}}i + \frac{-42}{\sqrt{65}}j$
or
$\frac{24}{\sqrt{65}}i - \frac{42}{\sqrt{65}}j$

Lessons 21-33, All of Sections 7.6, 8.1, 8.2, 8.3, and 8.4

10. Approximate the magnitude of the vector $c = -9i + 4j$ to the nearest tenth.

A. $\|c\| = 8.4$

B. $\|c\| = 9.8$

C. $\|c\| = 8.7$

D. $\|c\| = 9.6$

E. None of the above

$\|c\| = \sqrt{9^2 + 4^2}$

$= \sqrt{81+16} = \sqrt{97}$

≈ 9.8485

11. Approximate smallest positive angle θ between the positive x-axis and vector $c = -9i + 4j$ to the nearest tenth of a degree.

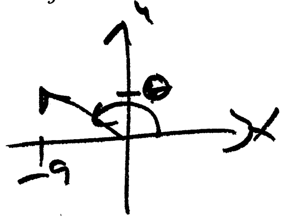
A. $\theta = 114.0°$

B. $\theta = 109.7°$

C. $\theta = 156.0°$

D. $\theta = 160.3°$

E. None of the above



$\tan\theta = \frac{4}{-9}$

$\theta = \tan^{-1}(-4/9) + 180°$

$\theta = -23.9625° + 180°$

$\theta = 156.0375°$

12. Find the angle between the two vectors to the nearest tenth of a degree.

$a = \langle -1, 7 \rangle, b = \langle 2, 15 \rangle$

Not on Fall '14 Ex. 3

A. $\theta = 64.2°$

B. $\theta = 15.7°$

C. $\theta = 25.8°$

D. $\theta = 74.3°$

E. None of the above

$\|a\| = \sqrt{1^2 + 7^2} = \sqrt{50}$

$\|b\| = \sqrt{2^2 + 15^2} = \sqrt{229}$

$a \cdot b = (-1)(2) + (7)(15)$

$= -2 + 105 = 103$

$\cos\theta = \frac{103}{\sqrt{50}\sqrt{229}} = \frac{103}{\sqrt{11450}}$

13. Forces $F_1 = \langle -2, 1 \rangle, F_2 = \langle 1, 5 \rangle, F_3 = \langle 4, -11 \rangle$ act at a point P. Find an additional force $\boldsymbol{G}$ such that equilibrium $(\langle 0, 0 \rangle)$ occurs.

A. $G = \langle -3, 5 \rangle$

B. $G = \langle -4, 8 \rangle$

C. $G = \langle -6, 9 \rangle$

D. $G = \langle -1, 6 \rangle$

E. None of the above

$F_{net} = \langle -2+1+4, 1+5-11 \rangle$

$F_{net} = \langle 3, -5 \rangle$

$+ G = \langle -3, 5 \rangle$

$\overline{\langle 0, 0 \rangle}$

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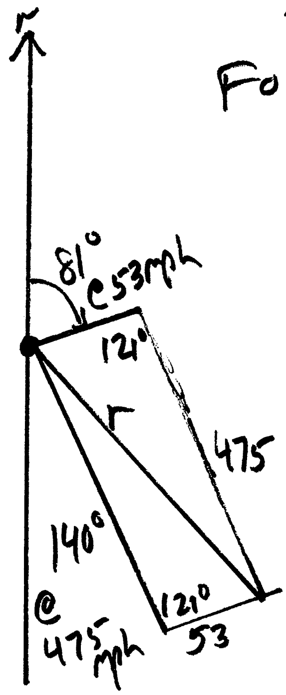
Lessons 21-33, All of Sections 7.6, 8.1, 8.2, 8.3, and 8.4

Questions 14 and 15: An airplane is flying in the direction 140° with an airspeed of 475 mph and a 53 mph wind is blowing in the direction 81°.



For ▱ method: 1. $140° - 81° = 59°$ 2. $180° - 59° = 121°$

For vector method:

$P: \langle 475\cos 140°, 475\sin 140° \rangle$

$+ w: \langle 53\cos 81°, 53\sin 81° \rangle$

$r: \langle -355.5801, 357.67167 \rangle$

14. Approximate the ground speed of the airplane to the nearest mile per hour.

A. 478 *mph*

B. 450 *mph*

C. 497 *mph*

D. 504 *mph*

E. None of the above

▱ method $r^2 = 475^2 + 53^2 - 2(475)(53)\cos 121°$

$\|r\| = 504.3473$

Vector method $\|r\| = \sqrt{x^2+y^2} = \sqrt{(-355.6)^2 + (357.7)^2}$

$\|r\| = 504.3473$

15. Approximate the true course of the airplane to the nearest degree.

A. 139°

B. 146°

C. 135°

D. 143°

E. None of the above

▱ method: $\dfrac{\sin\alpha}{475} = \dfrac{\sin 121°}{504.3473}$

$\sin\alpha = \dfrac{475\sin 121°}{504.3473} = 0.8073$

$\alpha = \sin^{-1}(0.8073) = 53.8320°$

True Course: $81 + 54° = \underline{135°}$

Vector method

r is in QII (airline)

$\tan\theta = \dfrac{y}{x} = \dfrac{357.7}{-355.6} = -1.0059$

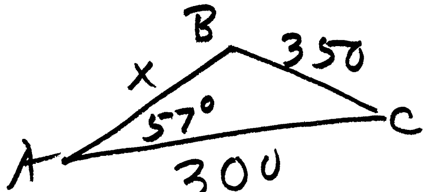
$\theta = \tan^{-1}(-1.0059) + 180°$

$\theta = -45 + 180° = \underline{135°}$

Lessons 21-33, All of Sections 7.6, 8.1, 8.2, 8.3, and 8.4

16. To determine the distance between two points A and B, a surveyor chooses a points C that is 300 yards from A and 350 yards from B. If $m\angle BAC = 57°$, approximate the distance between A and B to the nearest whole number.

A. 461 yards
B. 426 yards
C. 495 yards
D. 407 yards
E. None of the above



$$\frac{\sin 57^\circ}{350} = \frac{\sin \beta}{300}$$

$$\sin\beta = \frac{300 \sin 57^\circ}{350}$$

$$\sin\beta = 0.7189$$

$$B = 45.9605°$$

$$\gamma = 180° - (57° + 45.96°)$$

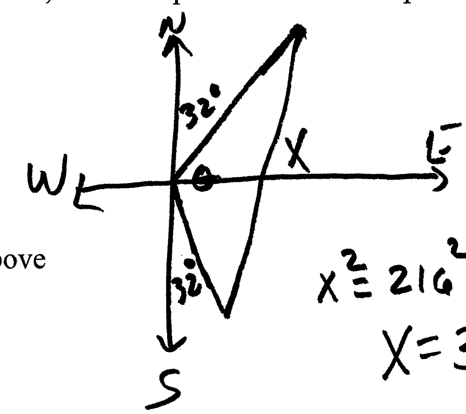
$$\gamma = 77.0395°$$

$$\frac{\sin 57^\circ}{350} = \frac{\sin 77.04^\circ}{x}$$

$$x = 406.6957$$

17. A ship leaves port at 1:00 pm and travels $N32°E$ at a rate of 54 mph. At the same time a second ship leaves the same port and travels $S32°E$ at a rate of 41 mph. To the nearest mile, how far apart are the two ships at 5:00 pm? $D = rt$

A. 323 miles
B. 348 miles
C. 286 miles
D. 264 miles
E. None of the above



$D_1 = 54(4) = 216\text{mi}$

$D_2 = 41(4) = 164\text{mi}$

$\theta = 180° - (32° + 32°) = 116°$

$x^2 = 216^2 + 164^2 - 2(216)(164)\cos 116°$

$x = 323.4343$

18. Find a vector with 6 times the magnitude in the same direction of vector $a = \langle -3, 7 \rangle$

A. $\left\langle \frac{18}{\sqrt{58}}, \frac{-42}{\sqrt{58}} \right\rangle$

B. $\langle -18, 42 \rangle$

C. $\left\langle \frac{-18}{\sqrt{58}}, \frac{42}{\sqrt{58}} \right\rangle$

D. $\langle 18, -42 \rangle$

E. None of the above

$6a = 6\langle -3, 7 \rangle$

$= \langle -18, 42 \rangle$

Had it Asked ... A vector with magnitude of 6 ... you would find $6\hat{u}$

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Lessons 21-33, All of Sections 7.6, 8.1, 8.2, 8.3, and 8.4

Question	Answers	
1.	$\frac{-\pi}{3}$	A
2.	$\frac{119}{169}$	D
3.	$1.2256, 0.6235$	C
4.	497	D
5.	Larger $\beta = 93.1°$	B
6.	$c = 28.3$	E
7.	$96.4°$	C
8.	$\langle 23, -15 \rangle$	D
9.	$\frac{24}{\sqrt{65}} i - \frac{42}{\sqrt{65}} j$	A
10.	$\lVert c \rVert = 9.8$	B
11.	$\theta = 156.0°$	C
12.	$\theta = 15.7°$	B
13.	$G = \langle -3, 5 \rangle$	A
14.	$504\, mph$	D
15.	$135°$	C
16.	407 yards	C
17.	323 miles	A
18.	$\langle -18, 42 \rangle$	B