MA 161M EXAM 1 Sept. 20, 2002

Name Key

Note: If work is not shown, no credit will be given. NO CALCULATORS.

(10 pts) 1) Find an equation of the line perpendicular to $2x + y + 3 = 0$ and passing through the point $(-1, 1)$.

Equation of given line can be written as $y = -2x - 3$

$\therefore$ Line $\perp$ to given line has slope $\frac{1}{2}$

Since Line $\perp$ " " " passes through $(-1,1)$ it has

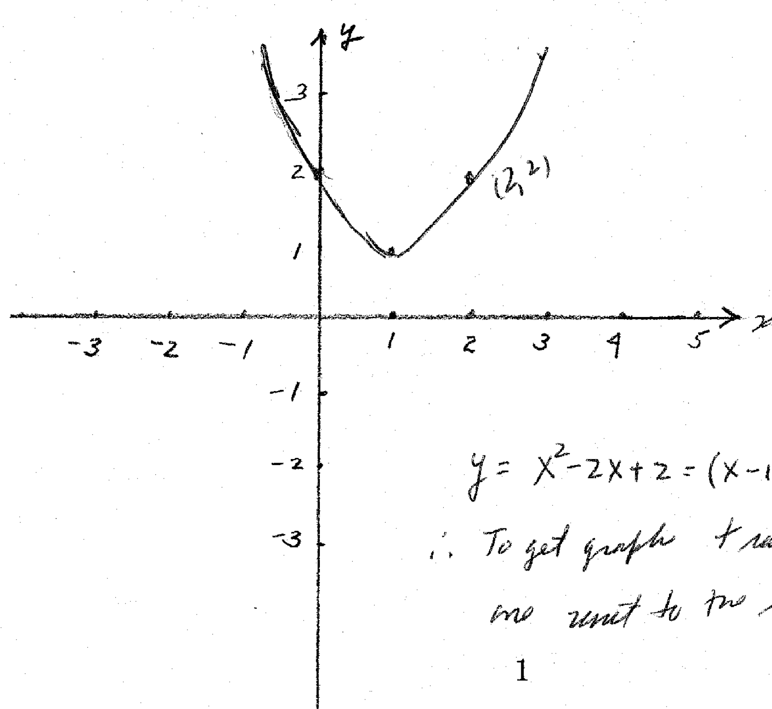
eqn $(y-1) = \frac{1}{2}(x+1)$

or $y = \frac{1}{2}x + 3/2$

or $2y - x - 3 = 0$

ANSWER $y = \frac{1}{2}x + 3/2$ or $2y - x - 3 = 0$

(10 pts) 2) Sketch the graph of $y = x^2 - 2x + 2$ without plotting points. State the steps you used to get your graph from the graph of a simple function.



$y = x^2 - 2x + 2 = (x-1)^2 + 1$

$\therefore$ To get graph translate graph of $y = x^2$ one unit to the right and one unit up

(7 pts) 3) (a) Find a formula for the inverse of the function

$$f(x) = \frac{1+2x}{3+4x} \qquad x \neq -3/4$$

(3 pts) (b) What is the domain of the inverse function?

Solve $y = \frac{1+2x}{3+4x}$ for x & get $f^{-1}(y)$

$y(3+4x) = 1+2x$

$3y-1 = (-4y+2)x$

or $x = \frac{3y-1}{2(-2y+1)}$ so $x = f^{-1}(y)$

Relabeling to make x as the independent variable gives

$y = \frac{3x-1}{2(-2x+1)}$ $x \neq \frac{1}{2}$

INVERSE $y = \frac{3x-1}{2(-2x+1)} = \frac{3x-1}{2-4x}$ DOMAIN All $x \neq 1/2$

(10 pts) 4) Let

$$f(x) = \begin{cases} x+c & \text{if } x \leq 3 \\ cx^2 - 2 & \text{if } x > 3 \end{cases}$$

For what value of c, if any, is the function continuous at $x = 3$? Why?

For f to be continuous at c we require $f(3) = \lim_{x \to 3+} f(x) = \lim_{x \to 3-} f(x)$

Since $\lim_{x \to 3-} f(x) = 3+c$ and $\lim_{x \to 3+} f(x) = 9c-2$

we get that c must satisfy

$3+c = 9c-2$

or $5 = 8c$

$\therefore c = 5/8$

ANSWER $5/8$

5) Find the following limits or show that they do not exist.

(7 pts) (a) $\lim_{x\to\infty} \dfrac{7x^3+2x+100}{8x^3+12x^2+2x+1}$

$$\frac{7x^3+2x+100}{8x^3+12x^2+2x+1} = \frac{\cancel{x^3}}{\cancel{x^3}} \frac{\left[7+\frac{2}{x^2}+\frac{100}{x^3}\right]}{\left[8+\frac{12}{x}+\frac{2}{x^2}+\frac{1}{x^3}\right]}$$

0 0 0 0 0 as $x\to\infty$

Thus

lim

ANSWER $7/8$

(6 pts) (b) $\lim_{x\to -4} \dfrac{x^2+x-12}{x+4}$

$$\frac{x^2+x-12}{x+4} = \frac{\cancel{(x+4)}(x-3)}{\cancel{x+4}} = (x-3) \qquad x \neq -4$$

$$\therefore \lim_{x\to -4} \frac{x^2+x-12}{x+4} = \lim_{x\to -4} (x-3) = -7$$

ANSWER -7

(7 pts) (c) $\lim_{x\to 0} |x| \cos \dfrac{\pi}{x^2}$

$$-1 \le \cos \pi/x^2 \le 1$$

$$\therefore -|x| \le |x| \cos \pi/x^2 \le |x|$$

Since $|x|$ and $-|x| \to 0$ as $x\to 0$, by the Squeeze Theorem, so does $|x|\cos \pi/x^2$

ANSWER 0

NOTE: IN PROBLEMS 6 AND 7 YOU CANNOT USE THE DIFFERENTIATION RULES OF CHAP. 3.

(10 pts) 6) Find the slope of the tangent line to the graph of

$$y = \frac{1}{x+1} \quad x \neq -1$$

at the point with x coordinate $= 2$.

Slope of tangent line $= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

$= \lim_{h \to 0} \frac{1}{h}\left[\frac{1}{2+h+1} - \frac{1}{(2+1)}\right] = \lim_{h \to 0} \left[\frac{(2+1) - ((2+1)+h)}{(2+1)(2+h+1)}\right]\frac{1}{h}$

$= \lim_{h \to 0} \frac{-h}{(2+1)(2+h+1)}\frac{1}{h} = -\frac{1}{(3)^2} = -\frac{1}{9}$

ANSWER $-\frac{1}{9}$

7) Let $f(x) = \sqrt{2+x}$

(5 pts) (a) What is the domain of f?

(10 pts) (b) Find $f'(x)$. $f'(x) = \lim_{h \to 0}\left[\frac{\sqrt{2+(x+h)} - \sqrt{2+x}}{h}\right]$

$= \lim_{h \to 0} \frac{1}{h}\left[\sqrt{2+(x+h)} - \sqrt{2+x}\right]\frac{\sqrt{2+(x+h)} + \sqrt{2+x}}{\sqrt{2+(x+h)} + \sqrt{2+x}} = \cancel{\lim_{h \to 0} \frac{1}{h}}$

$= \lim_{h \to 0} \frac{1}{h}\left[\frac{2+(x+h)-(2+x)}{\sqrt{2+x+h} + \sqrt{2+x}}\right] = \lim_{h \to 0} \frac{1}{\cancel{h}}\left[\frac{\cancel{h}}{\sqrt{2+x+h} + \sqrt{2+x}}\right] = \frac{1}{2\sqrt{2+x}}$

DOMAIN $x \geq -2$ $f'(x) = \frac{1}{2\sqrt{2+x}}$

(15 pts) 8) Show that the equation $x^5 - 2x^2 + 1 = 0$ has at least one negative root. What theorem justifies your conclusion?

By intermediate value theorem if $f(x) = x^5 - 2x^2 + 1$
is positive at some integer k and negative at $k-1$
(or vice-versa) then by the intermediate value theorem
there is a point in the interval $(k-1, k)$ at which
$f(x) = 0$, say $x = a$. Thus a is root of $x^5 - 2x^2 + 1 = 0$

Make table of values

x	$f(x)$
0	1
-1	-2

$\therefore$ By int value theorem $x^5 - 2x^2 + 1$ has a root between -1 and 0.